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Investigating students' heights through a data-modelling approach

Joanne Mulligan

joanne.mulligan@mq.edu.au

Macquarie University, Sydney, NSW, Australia

Melinda Kirk, Russell Tytler & Peta White

melinda.kirk@deakin.edu.au, russell.tytler@deakin.edu.au, peta.white@deakin.edu.au

Deakin University, Melbourne,Victoria, Australia

Maria Capsalis, Essex Heights Primary School, Melbourne, Victoria, Australia

Maria.Capsalis@education.vic.gov.au

This article illustrates one of several learning sequences from the Interdisciplinary Mathematics and Science (IMS) Learning project, connecting science and mathematics learning in the primary school. Investigating Body Height involved a series of investigations conducted in Grades 5 and 6 across six classes in one school. The project teachers implemented an enquiry-based pedagogical model to collect, organise, analyse and represent data about students' heights. Statistical ideas and various forms of graphical representation were developed well beyond curriculum expectations.

In this article we describe a learning sequence developed as part of an interdisciplinary mathematics-science research project, where each subject is mutually reinforcing and there is a strong focus on students inventing and refining representations. An authentic context was posed by the teacher to generate a mathematical investigation where students could productively engage in mathematics learning and support data-modelling skills common to science learning. Students were initially challenged to consider how many students in their Grade 5/6 class would meet the height requirements of a theme park ride. This challenge required students to estimate their own height and compare their height to their peers through direct comparison, informal and formal measures. Following this stage, comparison of estimates with actual measures and an extension to measuring the heights of their 'buddy' student, in their buddy Prep/Grade 1 class, were conducted. The collaborative process provided a social context for learning using an iterative pedagogical cycle and afforded multiple opportunities for students to refine and develop their measurement and data-modelling skills, representations and mathematical concepts.

The IMS pedagogical model

In our previous APMC article we described the pedagogical approach implemented by teachers in the project (Mulligan et al., xxxx, p.x). The approach works as a cycle (Tytler et al., 2021) and in this Investigating Body Height sequence teachers moved flexibly between Representational Challenges and the Building Consensus stages of learning.

The Learning Sequence: Student reasoning and representations through the pedagogical cycle

Orienting: Students were oriented to investigate their height, that of their classmates and their buddies in a lower grade level. The Orienting stage provided an authentic context and real-life investigation where students were challenged to answer this question:

Consider how many students in your grade could enter a theme park ride requiring a minimum person height of 1.4m?

The preliminary lesson purposefully oriented students to estimate their own height, considering whether they were taller or shorter than 140cm/1.4m. Students were challenged to make individual height estimates, share their estimates and suggest ways of representing the whole class data as a display.

Posing Representational Challenges

This stage required students to review and record their estimate and represent their individual height. Individual estimates were then collated and organised to form a class data display. Later students ordered the class estimates and then measured and represented their actual heights, using formal units of measure (millimetres/centimetres). Various forms of graphs were elicited and gradually refined following comparison and discussion about how to best illustrate these data (Mulligan, 2015). Figure 1 shows a student's recording of class estimates using a column graph. Figures 2a and 2b show more sophisticated graphs showing the combined class data.

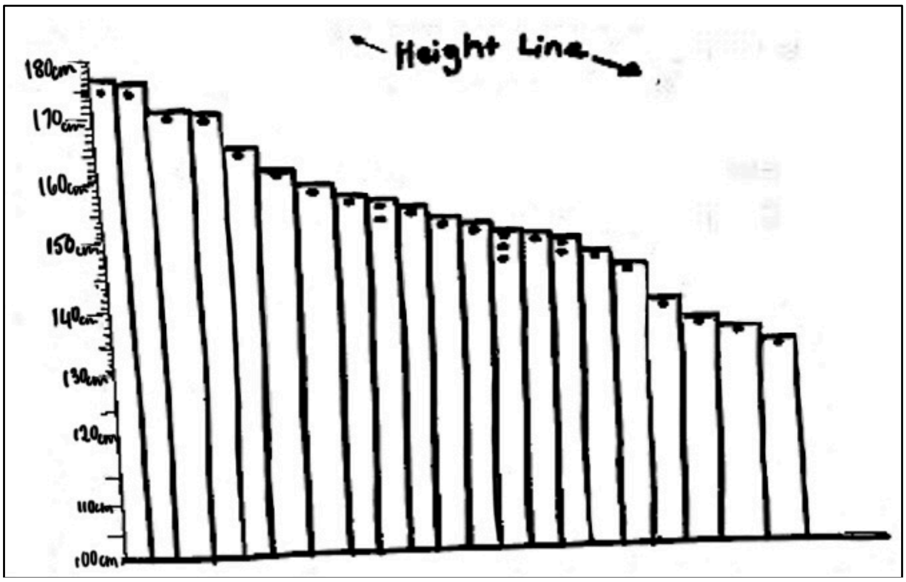

Figure 1. A student's invented column graph. Heights are ordered and represented by columns, using a scale of 10cm increments, with dots to indicate the number of students with the same estimated height.

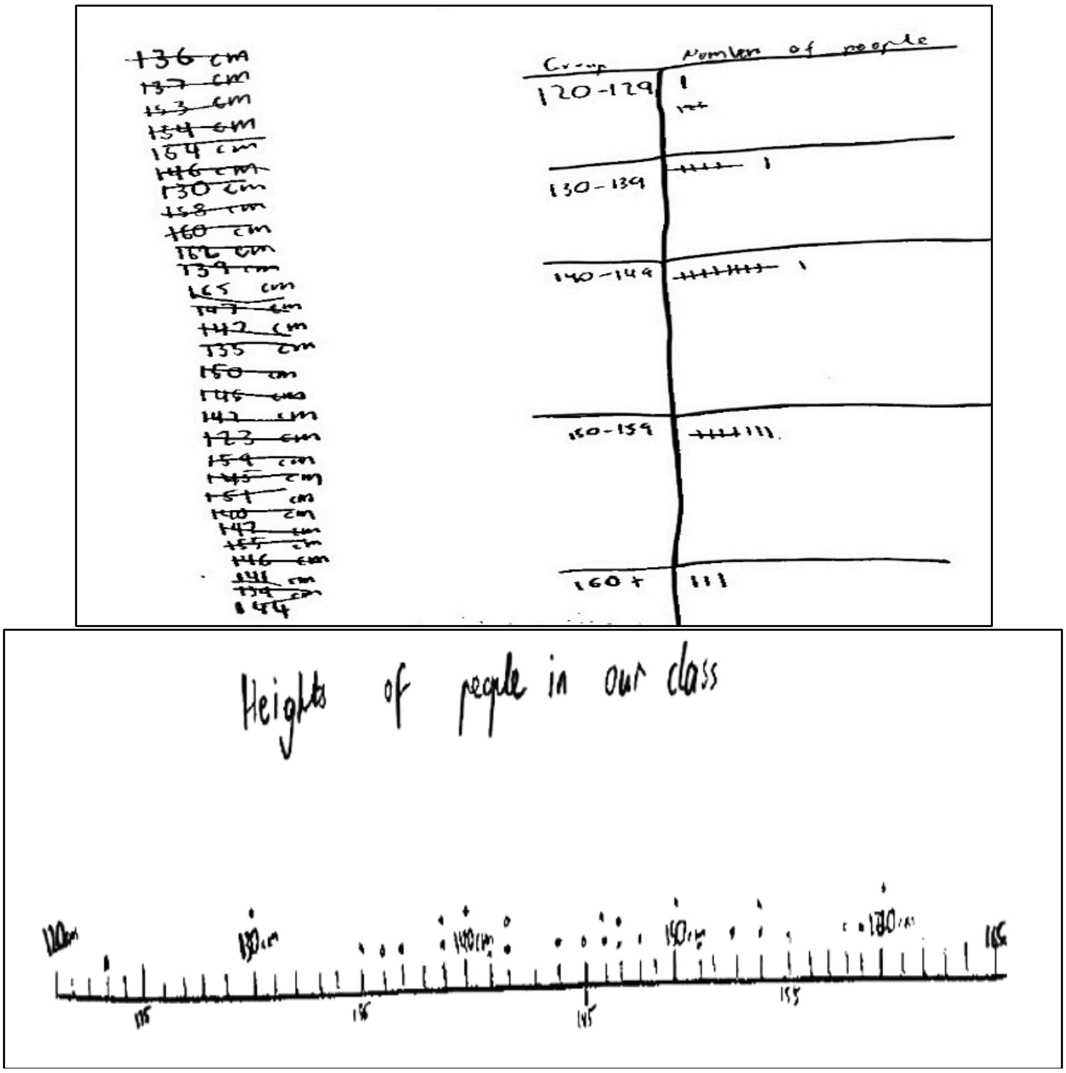

Figure 2a. and 2b. A student's recording of the whole class estimated heights in groups or clusters (Figure 2a) and their representation of the whole class data as a dot plot (Figure 2b). The student explained why they chose this recording and representation method.

> It makes it easier to know how many are in each sort of category for height. It makes it easier to find different things…I chose to do that…You can see the, every single height and the variation and the clustering is here…(pointing to approximately 146).

Figure 2b. This dot plot representation was purposefully selected by the teacher to discuss with the whole class. Students were prompted to consider if it was clear and easy to understand. The class identified that the dots were easy to see and understand, and the outlier height, 123cm, easy to identify. Class discussion about the range of heights naturally ensued, with the language of range, average, mode and median used in light of prior introduction of these concepts.

In Figure 3 the student makes a more sophisticated representation of estimates of class members' heights in clusters again raising discussion about mean, median and mode.

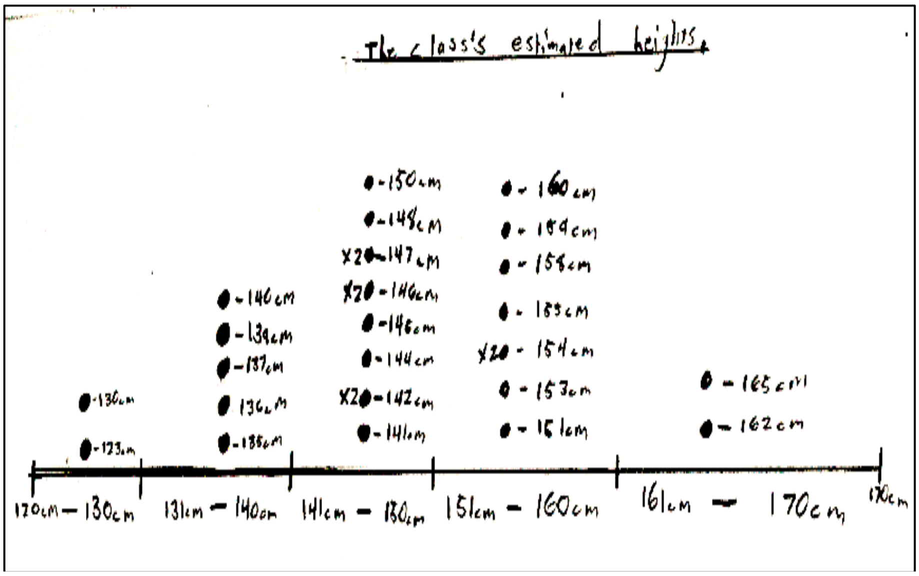

Figure 3. Grade 6 student's representation of estimated heights as clusters using a horizontal line plot with intervals shown in 10cm segments.

Building Consensus

In the Building Consensus stage students were encouraged to critically analyse the characteristics of their representations to determine effective and appropriate features. Teacher scaffolding enabled the productive review of student processes to develop an understanding of variability, and build on the ideas of range, mean and mode, and the representation of data as frequencies, categories and clusters.

Data modelling: Enabling the comparison of data sets

Students formally measured and compared their actual height with their estimates, by sharing their data to formulate a whole class data set. Students ordered the heights in varying ways to visually represent their individual recordings.

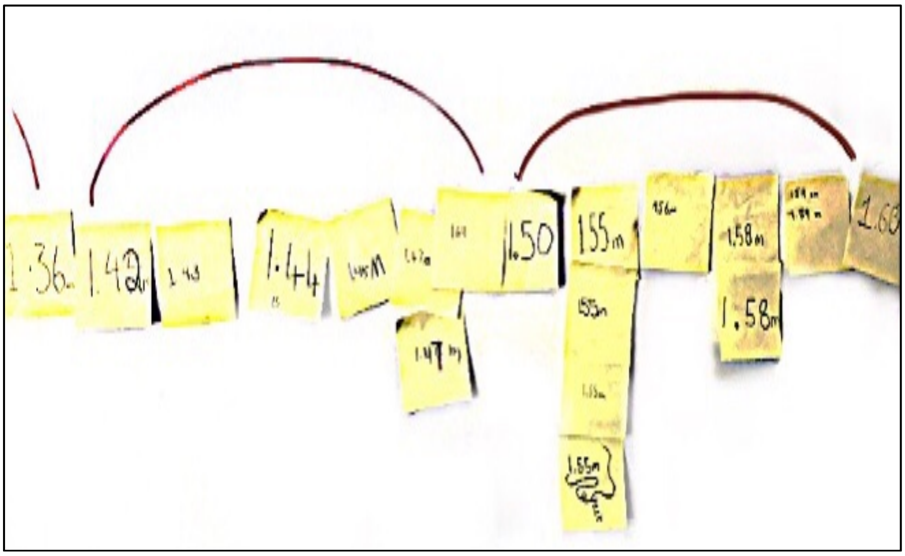

Figure 4. Students ordered their discrete recordings on post-it® notes to identify clusters.

The following teacher interview statement explains how the students moved from ordering their own heights by physically standing in groups of similar heights to forming groupings, or clusters of heights for the whole class data.

> I said, now have a think about how would you group them? …the second time they grouped them, they had like you know, 150 to 152. 153 to 155. So, it was just interesting to see that the visual representation developed their thinking of how to group a little more efficiently.

The students engaged in dynamic discussion around the exactness of some student height measurements, with students standing back to back and asking others to judge or confirm accuracy of height order. The need for consistent formal measurement naturally arose, with some students re-measuring to ensure accuracy.

Following whole class ordering and representations, students compared their actual heights with the class estimates (see Figure 5). Rather than being explicitly told, students were encouraged to invent ways of comparing the data. Column graphs, dot plots and other invented ways of visually comparing estimates and actual height data were explored.

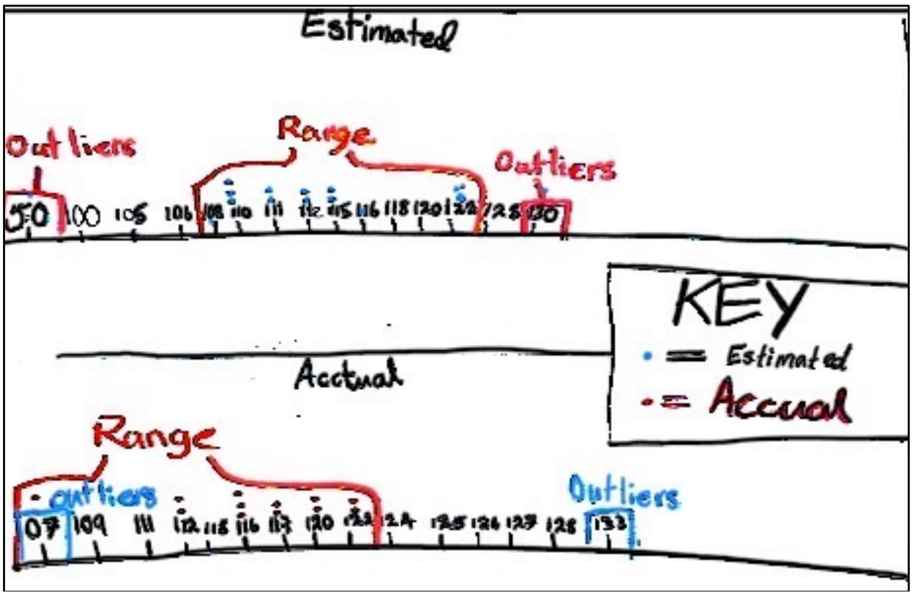

Figure 5. Dot plot of class estimate data and actual height data showing range and outlier measures.

Teachers recognised the need to identify effective student representations to review the data collaboratively as a whole class. The following teacher statement exemplifies and explains this process.

> I'd go around the class and see what evidence or what they were presenting in their book and then I'd take that book and highlight to the class. What were you thinking here as you were doing this? Like … can you explain what you're displaying here to the whole class? …and I tried to do that early on, like when they were doing their diagrams…so those students who were struggling to really represent their thinking could kind of get…maybe not copy that student's thinking but get an idea of what other people were kind of thinking that related to the data.

Students' negotiation and collaboration were indicative of students building consensus and conceptual understanding. The following teacher interview excerpt details deliberate guiding questions that were posed.

If someone else looked at this, what would they understand from your work? Are there things that you need to explain a bit better, or you know, show a bit more? That was a helpful tool for them, because then they went okay, and sometimes you'd even see them go and ask a friend, is this clear to understand? It's more student based. So, they would then do the thinking on it and kind of guide each other.

Data analysis, the identification of the shortest height, tallest height, range of heights and average height in each data set, naturally arose out of student interest and discussion, rather than from direct instructions from the teacher as a set of separate tasks.

Applying/extending understanding to new settings
The sequence provided multiple opportunities to apply concepts to related challenges. Students were challenged to record and share heights across classes in the same grade level. Then as a new challenge, students measured and recorded heights of their buddy class— Prep/Kindergarten or Grade One. They compared height data with their own to make inferences about the sampling and draw conclusions about student growth patterns over six years. An extension opportunity was also to record and compare individual arm length/foot size with heights, to identify possible inter-relationships.

Students elected to look for patterns and/or differences across classes or grade levels. One Grade 5 class reasoned that they should guess their peers' heights without looking at them. They called this 'Guess 1' and then by looking at them, 'Guess 2', before actually measuring them. This is shown in Figure 6 below.

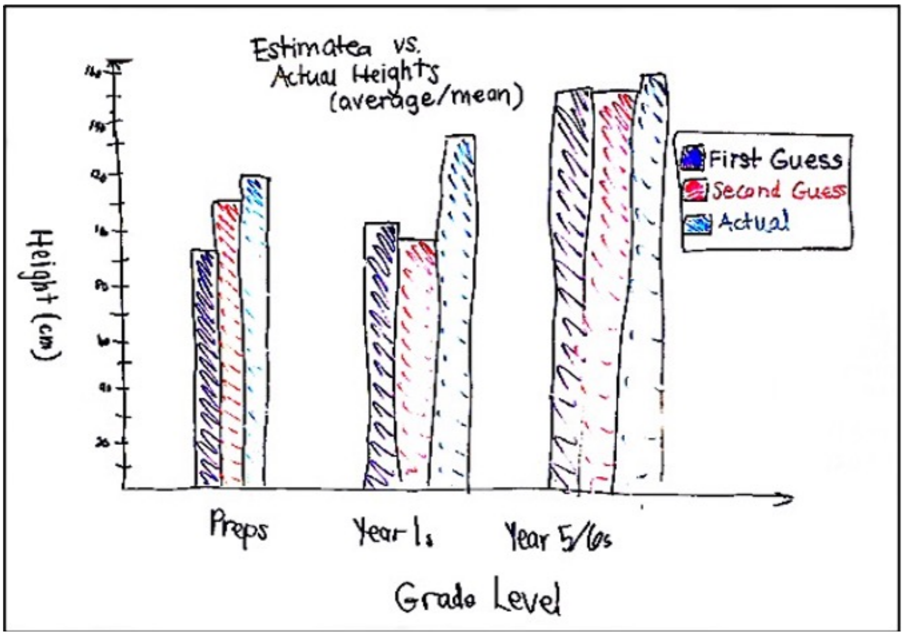

Figure 6. Representation of students' guesses and actual measurements across three cohorts: Prep, Grade 1 and Grade 5/6.

Through this iterative cycle of estimating, recording, representing and comparing different heights, students were afforded multiple opportunities to refine and develop their graphing skills. Students increasingly represented their data as dot plots, during class discussions and the building consensus stage.

As one student explained they had realised the benefits of using a dot plot when representing whole class data.

Yeah, like usually when we have a big dataset, I would use a dot plot cause it's like a lot more efficient, I guess. And then when it's just a small dataset or like we are comparing something with…I would like a side-by-side graph or a column.

Another student explained that they had changed their representations to be dot plots as a result of building consensus.

It gives you ideas and ways to improve on your own work…Yeah. I just…like for…the dot plot was actually the second one I did… I think it was the second week and we decided to experiment on that and because I did it. One person did it really well, and Miss thought it was really effective.

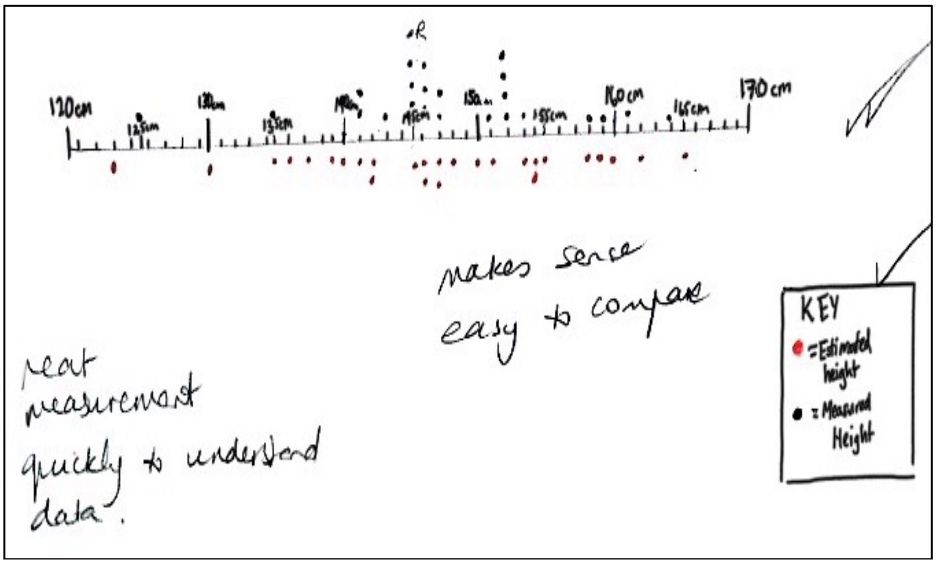

Figure 7. Student variations of the dot plot, representing and comparing estimated heights and measured heights above and below the line.

In the below dialogue with the class teacher the student reasoned why they chose to represent the data in this way during the building consensus stage.

Teacher (T): So do you think these colours could be on top as well? (teacher pointing to the data under the number line)
Student (S): Yes
T: Why did you think of this (point to height dots under the number line?)
S: I put them (estimated heights) down there, so it is easy to compare.
T: Yes, it's easy to compare. That's the good thing about it. I also like how you've got a key; I like how it is neat; I like how you've used measurements and I really like how I can look at this quickly and am able to understand the data.

Participating teachers commented on the benefits of combining meaningful guiding questions with purposeful representations to enable student understanding. They also commented on how students' graphing skills developed not only in learning how to draw a graph, but the purpose of using a graph.

They really enjoyed that ability to either use a dot plot or side by side graph. You know, they expanded their knowledge of the different graphs and the best way to show…they were actually analysing the uses of those graphs instead of just what we normally do, give them a table of data and say, you're going to make this graph today. So, it actually got them thinking about why they would choose a graph and whether it represented their information well.

Implications for the mathematics classroom

The Investigating Body Height learning sequence demonstrated a range of possibilities for mathematics learning supporting the Australian Curriculum-Mathematics, particularly the reasoning and problem-solving aspects of the Proficiencies (Australian Curriculum, Assessment & Reporting Authority, 2018). Student learning through purposeful invention, comparison and refinement processes were illustrated in the students' explanations and representations of data enabled through teacher scaffolding and the building consensus process. Through situating mathematics in meaningful investigations, the purpose of measuring and data-modelling processes were realised. One teacher commented: "They (the students) were actually thinking through what the purpose was and how they could best show that…which is what you want them to do with data instead of just you know, doing what they're told to do". The open-ended challenge and promotion of student invention allowed variation in student responses and teacher insight into student understanding of statistical concepts as well as number and measurement skills. One Grade 5 teacher described the pedagogical approach and learning sequence design as 'differentiation'. They noted that all students were able to participate at their own level and that teachers were able to identify gaps in students' knowledge and skills through their representations and explanations, no matter how basic or informal.

The teaching of length in the upper primary grades is often focused on calculating and applying known formal measures to problems that are not based on real-life contexts and are often devoid of any actual measurement by the students. Allocating sufficient time to such extensive explorations is perhaps very challenging given pressured timetables and the crowded curriculum. What this article illustrates is that there are multiple opportunities to develop students' number, measurement and data-modelling skills at the same time— as well as learning about statistical concepts and representational processes that can feed into science investigations. These outcomes can be achieved beyond curriculum expectations if such problem contexts can be explored systematically over a series of lessons. The challenge is to prioritise and carefully plan these kinds of integrated mathematical tasks to promote more relevant and sophisticated mathematical thinking at the primary school level.

Acknowledgments
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