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Linking Mathematics and Science Productively
Key questions

1. What are the key conceptual and curricular design features of these interdisciplinary learning sequences that support foundational learning in each discipline?

2. What are the design challenges associated with working with teachers to achieve this interdisciplinary alignment?

3. What pedagogical design principles and strategies underpin effective enactment of such a curriculum?
ENRICHING MATHS AND SCIENCE LEARNING: AN INTERDISCIPLINARY APPROACH

This international, longitudinal project aims to investigate the effectiveness of an innovative interdisciplinary learning approach in mathematics and science. Through collaborating primary schools in Australia and the United States of America (USA), it will investigate how students' invention and transformation of representational systems can connect to support deeper reasoning and learning. The project will form the bases for new curricular designs that leverage students' representational practices across science, technology, engineering and mathematics (STEM) disciplines to promote more robust and generative knowledge.

https://imslearning.org/

Background to the IMS pedagogy

Learning as induction into the multi modal discursive practices of science and mathematics (Latour, Peirce, Lemke)

Model based reasoning, sociosemiotic perspectives (Lehrer & Schauble, Lemke)

Pedagogy: guided inquiry where children generate data/observations and invent, compare, assess and revise, and coordinate representations.

Maths and science interact productively, each raising questions that advances the other. There is a focus on constructs that are common to both.

Representational tools are crucial resources for speculating, reasoning, contesting and justifying explanations, knowledge building, and communicating.
The key principles are that
a) the mathematics and science concepts and practices are mutually reinforcing,
b) the interdisciplinary tasks lead to fresh learning in each subject, and not simply application of known procedures and concepts, and
c) that the reasoning and learning in each subject reflects disciplinary knowledge building practices, involving the invention and guided refinement of representational systems.
## The IMS topics

<table>
<thead>
<tr>
<th>Topic (predominantly science)</th>
<th>Grade level</th>
<th>Mathematics concepts and practices</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chemical science: Dissolving and mixing, chemical change, cooking.</td>
<td>2</td>
<td>Representing time sequences of mixing and dissolving under different conditions, timing cooking- measurement for cooking recipes: standard and informal units, fractions, proportion.</td>
</tr>
<tr>
<td>Astronomy: Shadows, sun movement, explaining day and night: earth and space perspectives.</td>
<td>1, 4</td>
<td>Angle as rotation and length of shadow (formal and informal measures), graphing shadow length, time sequencing, perspective taking.</td>
</tr>
<tr>
<td>Ecology: Living things, diversity, distribution and adaptive features related to habitat.</td>
<td>1, 4</td>
<td>Variation, data modeling of living things in sample plots, tables and graphs, space and mapping, measurement, area, coordinates, directionality, sampling, using a scale.</td>
</tr>
<tr>
<td>Astronomy: Solar system, day and night, planetary features, moon movement and phases.</td>
<td>5/6</td>
<td>Conceptualising ratio of planetary size and distance, angle measurements for moon observations, compass points, tracking position over time, perspective taking from earth and space, representing cosmological distances: powers of ten.</td>
</tr>
<tr>
<td>Plant growth: plant structure and function, life cycle, growth needs and patterns.</td>
<td>2</td>
<td>Measurement of plant height, leaf size, shape and number, tracking growth over time, tables, line graphs, units (cm and mm- formal units), time intervals, using a scale.</td>
</tr>
<tr>
<td>Measurement (height): height measure and variation, differences between populations.</td>
<td>5/6</td>
<td>Measurement (m and cm), data modeling, variation, measures of central tendency and of variation, comparison through graphs, categorizing/organizing data, sampling.</td>
</tr>
<tr>
<td>Flight: Paper Helicopters: Flight and air flow, modeling and design.</td>
<td>2</td>
<td>Measure of Paper Helicopter parameters, time, data variation, data modeling, number line.</td>
</tr>
<tr>
<td>Motion: Representing speed, distance and time relations, constant speed, acceleration.</td>
<td>1, 4</td>
<td>Embodied representation of relation between distance, time and speed, length measures, modeling variation, graphing speed, distance, time, distance-speed-time relations for motion down a ramp.</td>
</tr>
<tr>
<td>Water: personal use and conservation of water.</td>
<td>2</td>
<td>Estimation and measure of water use, informal and formal measures of volume.</td>
</tr>
<tr>
<td>Light: vision, reflection and image creation.</td>
<td>4/5</td>
<td>Angle, rotation, reflection, directionality, symmetry.</td>
</tr>
</tbody>
</table>
Sequences have been designed in consultation with teachers and have been implemented in schools, supported by regular review and planning sessions.

<table>
<thead>
<tr>
<th>Interdisciplinary Maths and Science Resources</th>
</tr>
</thead>
<tbody>
<tr>
<td>These resources are the teaching and learning sequences developed through this research.</td>
</tr>
<tr>
<td>You can download and adapt these sequences. They include students' work samples to help you anticipate student representational productions, and illustrate the learning.</td>
</tr>
<tr>
<td>If you try these resources, either as sequences or discrete activities, we would love to hear from you about your experience.</td>
</tr>
<tr>
<td><strong>Astronomy – Day and Night (Year 1)</strong></td>
</tr>
<tr>
<td><strong>Ecology: What Lives Here? (Year 1)</strong></td>
</tr>
<tr>
<td><strong>Push and Pull – Paper Helicopters (Year 2)</strong></td>
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<tr>
<td><strong>Heat and Temperature (Year 3)</strong></td>
</tr>
<tr>
<td><strong>Investigating Our Height – Body Maths (Year 5)</strong></td>
</tr>
<tr>
<td><strong>Light and Properties (Year 5)</strong></td>
</tr>
</tbody>
</table>
1. Orienting:
   Establishing what’s worth noticing

2. Posing Representational Challenges:
   Challenging students to explore and represent ideas and practices

3. Building consensus:
   a) Evaluating and synthesising student ideas and representations to reach agreement
   b) Refining and consolidating representations and concepts.

4. Applying and extending conceptual understanding:
   Posing further Representational Challenges (new modes, new contexts, or new concepts)
Exemplar Sequence

Year 2

Paper Helicopter Flight
Science Concepts
• Forces due to air flow, uplift and gravity
• Paper Helicopter design: variations in wing length, shape and weight and their impact on flight

Mathematics
Measure and timing (seconds and decimal places)
• Estimation and the importance of fine grain measure/timing
• Recording and representing data (tables and timelines/numberlines)
• Ordering and interpreting times (fastest/slowest and why)
• Comparing and relating time to the speed of the helicopter (meaning making – linking to scientific reasoning)
Recording of the drop times of paper helicopters, with different weights and student own designs. Looking at the variation in times, representing recordings and making inferences and drawing conclusions.

**Student Representational Focus**
Comparing displays, students learning about paper helicopter flight, and about effective data displays

- How to represent the data in order to make a decision about a valid measure of the time?
- Compare the different displays – what sort of things are different displays show? What do they show and what do they hide?
- What is the ‘best’ measure of the time of drop?
- What is the cause of the variation? How can this be shown?

**Teacher responsiveness and flexibility**

- What discussions would be important to have when teaching this?
- Productively drawing on student work - Building Consensus stage of IMS Pedagogy

[https://video.deakin.edu.au/media/t/1_zmosnhbg](https://video.deakin.edu.au/media/t/1_zmosnhbg)

[https://docs.google.com/document/d/1661ZuMKIDZ2I5uRoEKi3X0nYZTZL6cLcZdjWfyF465k/edit?usp=sharing](https://docs.google.com/document/d/1661ZuMKIDZ2I5uRoEKi3X0nYZTZL6cLcZdjWfyF465k/edit?usp=sharing)
Grade 2 paper helicopter
Lesson 1: What effects the fall of a Paper Helicopter?

Whole class predictions

Predictions
- fall straight down - paper might be too light
- fall slowly - it will spin and catch the air
- blades with catch the air
- paper clip will make it fall faster - heavier

Whole class student derived data representation

Focusing on variation
Whole class shared data identifying the fastest and slowest fall time of a standard Paper Helicopter

Teacher posed question
T: “Where do we start our numberline? Steven come and show us what to do”
Lesson 1: Individual student examples of data representations and focus on variation

Student-constructed horizontal timeline showing data ordered from the smallest to the biggest.

Student example, from board, of a timeline. Student identification of ‘the most’ frequency of helicopter speeds – 1.9 and 2.0 seconds.

Example of student recording of ‘tests’ and moving towards time-line representation.

Student-constructed horizontal timeline indicating ordered trial times. A question is presented at the top with a written response to the teacher’s questions at the bottom.
Lesson 1: Individual student examples of diagrammatic explanations

Examples of innovative student representations
Note the use of arrows in these examples to show rotation and complex air movement and air currents
Lesson 2: Investigating the effect of weight (different number of paperclips)

Examples of students using different recording systems and methods

Boardwork with class results
Examples of student reasoning

Paper helicopter with added weight

Example demonstrating gravity and upward lift

Example demonstrating gravity, upward lift and spinning

Example demonstrating the helicopter spinning around it’s vertex, lift force and gravity
### Grade 2 paper helicopter
Investigating the different designs (L3)

<table>
<thead>
<tr>
<th></th>
<th>M+F</th>
<th>E+2</th>
<th>D+S</th>
<th>T,5,8A</th>
<th>N+M</th>
<th>V+I</th>
<th>N+L</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test 1</td>
<td>1.4</td>
<td>0.8</td>
<td>1.3</td>
<td>1.2</td>
<td>1.1</td>
<td>1.4</td>
<td>1.2</td>
</tr>
<tr>
<td>Test 2</td>
<td>1.3</td>
<td>1.1</td>
<td>1.5</td>
<td>1.6</td>
<td>1.1</td>
<td>1.3</td>
<td>0.8</td>
</tr>
<tr>
<td>Test 3</td>
<td>1.0</td>
<td>Median</td>
<td>0.8</td>
<td>Median</td>
<td>1.4</td>
<td>Median</td>
<td>0.9</td>
</tr>
</tbody>
</table>

Emily’s board work showing the class data displays, first in a table with the median value identified, then as a timeline.

If you make the wings longer the whirlybird will fall slower.

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Teacher scribed for student with language difficulties.
Evidence of student learning beyond expectations (student work, post-sequence assessment, interviews, teacher testimony)

- Students articulating opposing forces of gravity and uplift, and the effect of wing design on air flow
- Appreciation of variation and how to represent it, the reason for multiple trials, and choice of median/mode as representative
- Measurement with stop watch and representing numbers with two decimal places
- Creation of tables and number lines
Curriculum design principles

a) classroom activity is grounded in exploration of phenomena within engaging and authentic contexts that raise questions that can be investigated through both science and mathematics;

b) learning disciplinary practices and concepts in each discipline is driven by teacher-guided invention, evaluation and refinement of representations;

c) the two subjects are mutually reinforcing so that developing ideas in one subject leads to questions that drive and inform learning in the other; and

d) the learning in each subject is fresh, as ideas are invented, tested, and refined, in contrast to interdisciplinary approaches restricted to the application of prior knowledge and procedures.
Challenges and benefits

• It was easier to design from science topics and build the mathematics from that

• Teachers tended to see this as independent of their maths curriculum

• Interlinking was not straightforward- statistical concepts, measure and data were easier but we also covered fractions, number, angle ...

• Aligning the mathematics in conformity with curriculum progression expectations was difficult – this needed flexibility and we argue that informal treatment of the maths is valuable (e.g. number line, decimals)

Benefits

• Student learning beyond expectations

• Students learning the epistemic practices of mathematics and of science (measure, data modelling ... )

• Dealing with authentic problems in an investigative environment was engaging
Design challenges working with teachers

• Teacher expectations around curriculum constraints sometimes made them reluctant to push into the mathematics (e.g. decimals, timeline). However, teachers were often pleased to push beyond expectations, for instance with graphs, or fractions.

• Teachers needed to be convinced of the rationale for representational challenge, leaving options open for students. They did however grow in this:

  I think at the start, the ... team and I were like “Oh, I don’t think they can do that”. And, you know there have been some teachers who want to give templates out... But it’s been really interesting seeing how they (the students) can create their own. (Emily, Grade 1 teacher)

• Teachers were not always knowledgeable about aspects of the science or mathematics, for instance with the idea of measures of centre revealing different aspects of a data set, and discussion of the nature and causes of variation was a challenge.
Pedagogical design challenges

Teachers, in using the approach successfully, needed to:

a) elicit and monitor student representational work and strategically conscript/critique these to move class discussion and representational refinement in productive directions towards target concepts;

b) respect students’ ideas to encourage contributions (the helicopter timeline introduction);

c) build consensus through this cycle of invention/evaluation/refinement (e.g. of graph work); and

d) extend representations and ideas to new representational forms
Start of student quotes as possible additions
Students were able to articulate the opposing forces of gravity, varied through paperclips, and air on the wings which was variously described as the wings ‘catching’ the air, the air getting stuck under the wings, the twirling creating an uplift, and the wind driving the spin.

*Gravity makes it goes down faster because it pulls down and the wind makes it go down slower because it is pulling up. But gravity has a bigger force, so it slowly(!) goes down.*

*Gravity is pulling it down and the wind is pulling up.*

*The helicopter works by you dropping the helicopter ... and the wind will whip it around while the gravity pulls it down.*
Spatial reasoning served as a both a mathematical and a scientific capability in interpreting the helicopter shapes, their symmetry, the movement of the helicopter and that of the air.

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*The shape of it* [affects the way it falls]. People cut out pieces, folded the edges so that had an effect on how long it took to catch the wind, that might have brought it down quicker or slower... we cut the wings in half and flipped them over, so we had four, two on each side, then we cut out triangles on each wing. That kind of changed the way the wind could go in, if air comes in really fast, if air goes up straight away when you drop it, it starts swaying slowly. But if the wind doesn’t ‘go in’ it falls straight to the ground.

The paperclips tug down on the helicopter. The weight makes it fall quicker (gives example and comparison to a basketball falling quicker than a ping-pong ball).

The longer winged one fell slower, because of the air, it takes a little more time for the air to catch it, because the wings are a bit longer... The faster it twirls, and the longer the wings, it can start to rise up... The weight will make it fall down quicker.

The longer wings are going to go slow and the shorter wings are going to go faster, because the air is going to get stuck up here (pointing to underneath the long wings) and here (pointing to small wings), they’re more small for the air to come out (point around the short wings).
Student recognition of a need for accurate measurements

To the assessment worksheet question:

Why did Ben and Tamara measure the drop 5 times instead of just one time?

20 students (66%) were explicit about the inevitability of variation in measure or the need to check and compare.

In interview, the suggested reason for repeat drops was identified as a means of ensuring more accurate recording. Repeat trials were described as needed for comparison purposes.

“to be more accurate”, “to make sure”

“allows for comparison” “so you can compare them”

“can tell if it’s the same, different or changed”
Student recognition of ‘best representative time’ or average

Students were able to recognise and use the median value of a set of numbers as being a fair representation of the data set, and to articulate the inevitability of variation on repeat measures, and suggest sensible reasons for this.

For the assessment worksheet question: What is your best guess for Tamara’s helicopter drop time?

6 students (20%) identified the median value. Most students identified the longest drop time as the ‘best guess’, reasoning in the interview that the goal was for the paper helicopter to fall slowly & the slowest was the ‘best helicopter’

In discussion during the interview, 13 of the 15 students chose either the median or the modal value as the ‘normal’ or the ‘main’, the ‘popular’, the ‘number in the middle’, the ‘most average’.

I’d choose 1.5 because I don’t want it to be too fast or too slow, so I’m choosing the ones that are perfect, not to high and not too low.

If it was 2.0 seconds it would be the highest amount of time but it was 1.0 second it would be the lowest. So, it wouldn’t be one of the two, it has to be directly in the middle.
Students valuing representing recordings in a timeline/numberline

In interview, students were asked to compare the interval timeline display to the ordered set of numbers, in terms of effectiveness for showing the results.

*The timeline shows how close the times are, and it helps you measure time, because it shows you how close the times are to all the other times.*

*I think (the table) doesn’t show how many times it happened clearly.*

*The number line is better, because you can actually see how many times a time was.*

*It’s way easier to understand than this (table) because you can tell how, there’s just one, but if there’s two you put two x’s, and it makes it a lot easier to understand.*

*You can see it more easily. The numbers are spread out and you can read it easily.*

*[The number line shows] how far they are apart, so these numbers are apart by one number, and this one is apart five numbers.*
Teacher challenges

Year 2 Curriculum alignment & mathematics
The measurement units, tools and methods of recording and analysis procedures were not always well aligned with curriculum expectations for this age grade.

Teachers were initially resistant to students timing the fall of helicopters with stopwatches (seconds, decimal places not Year 2 curriculum level)

Guided Inquiry & IMS Pedagogy
How to open up and honour, but critique student productions in ways that strategically guides understanding towards a group constructed consensus.

How to promote and productively use student ideas, whilst ensuring conceptual understanding.

How best to organise valid drop measures (two classes were initially held outside to allow a longer drop, but wind interfered rendering this location impractical. Teachers did not initially realise the importance of this)

How to establish a number line, created challenges that were not related to the interdisciplinary approach per se.

Interdisciplinarity
The mathematics-science interrelations required students to work with precise numbers which introduced students for the first time to the decimal notation.
Well, within a class you always have students working at different levels, but they can be extended and challenged, even when they’re a lower achieving student, or maybe they’re just not as great at presenting their work in writing, but they might be able to do it verbally or in a diagram. So, I think in terms of where they were, we definitely had a lot of extension, from the students in the middle and the top and the lower students as well were doing amazing things during the Unit and using vocab that would be beyond their level, drawing diagrams, and collecting data – with decimal points! And understanding what it means. So, I think it kind of shows that you can cater to everyone in the class and help everyone to grow.
Data representations
At the start it was tricky for them, collecting data and organizing it, but displaying their work with diagrams and graphs, they (representations) showed us what they could do and what we could help them with as well. Then, as the year progressed, I think they became more confident with collecting and reflecting on data and everything they’d done.
Learning from other student representations and comparative reasoning

_They are looking at other children’s work as well and then they realize, “Oh actually this person got a completely different measurement to me. What's different to the way I calculated it to the way the other person calculated it?”_ such as when we were doing the whirly birds, they were getting different timings and that's where it generated discussions between them, kind of like arguments.

_It’s like, "Oh you know you're dropping at a different height to me and of course your one hovers for longer" and it creates a lot of those science skills but also incidentally about learning math’s without calling the topic math._
Thank you

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