Investigating Our Height - Body Maths Year 5

How many students are tall enough to ride on theme park or water park rides requiring the minimum height of 1.4m?
Students investigate their height and that of their classmates. This learning sequence focuses on measurement of length/height in a familiar context and ways of representing data. Students estimate and order their heights using formal units (m/cm), develop understanding of variability, interpret ideas of range, mean, and mode and organise and represent data (categorical/cluster/frequency data). They apply concepts to new data sets.

INTERDISCIPLINARY MATHEMATICS AND SCIENCE (IMS) LEARNING

DEAKIN UNIVERSITY
MACQUARIE University
VANDERBILT UNIVERSITY
Australian Government
Australian Research Council

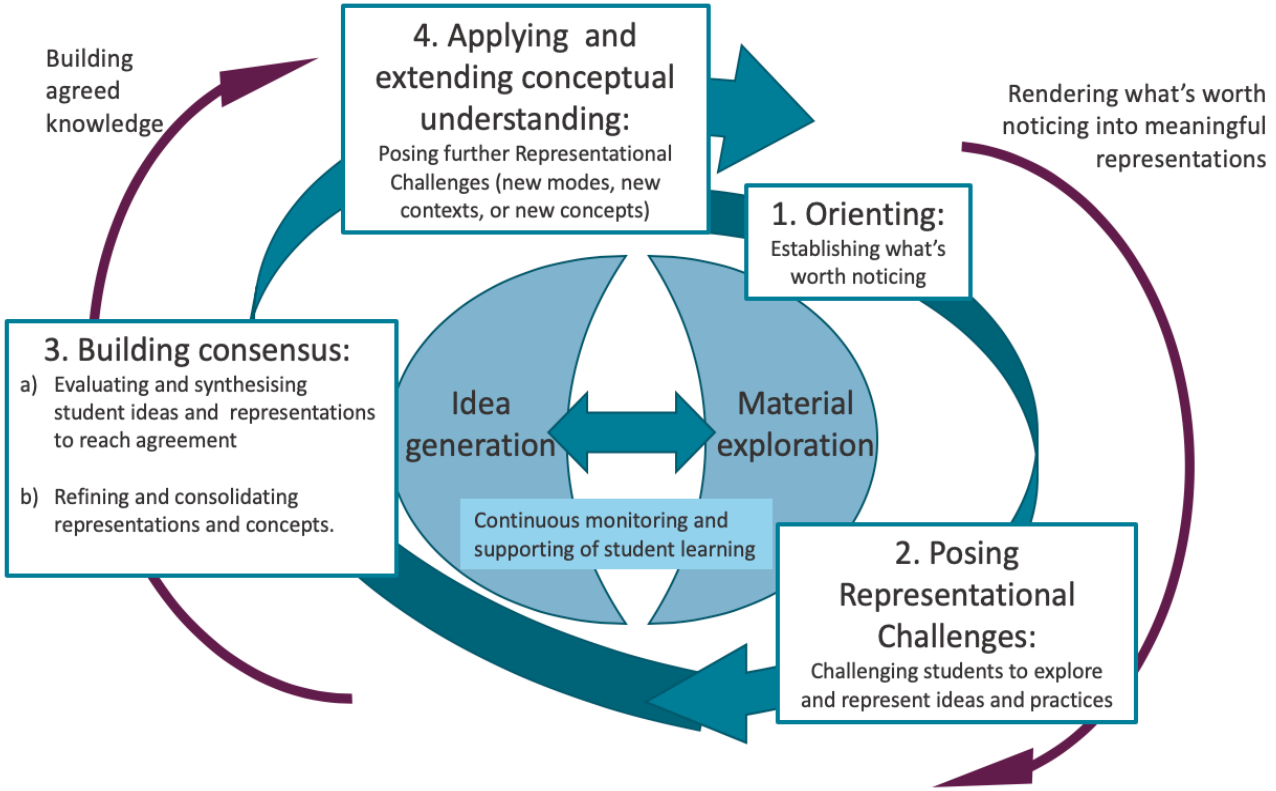
This teaching and learning sequence is one of a number that are designed to productively integrate mathematics with science, using a guided inquiry approach in which students construct, share, evaluate and revise multimodal representations to establish conceptual understanding. See website https://imslearning.org/

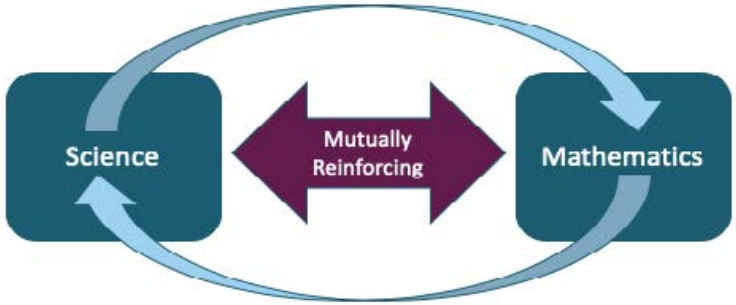
Interdisciplinary Mathematics and Science (IMS) Learning

IMS aims to enrich learning through two interconnected principles, which are key to the nature of the unit design and the pedagogy. The first principle concerns a focus on students constructing, evaluating, and refining multimodal representations, enacted through a four-stage IMS pedagogical model. The second principle concerns interdisciplinarity: the relation between science and mathematics. The project can be found at https://imslearning.org/
Below we describe the key features of the approach.

Student constructed representations

The teaching and learning sequences follow a guided inquiry pedagogy that focuses on students constructing, evaluating, refining, and extending multimodal representations. This is a literacy focus built on the insight that learning in both science and mathematics involves students being inducted into the representational practices that underpin explanation and problem solving. Representations can include diagrams, models, equations, graphs and tables, and symbols as well as written text. The approach involves a number of stages through which the teacher guides student learning. These stages, although distinct, often cycle and repeat within and across lessons. The model (to the right) showing these stages has been developed as an outcome of the IMS research.





Interdisciplinarity

In the teaching and learning sequences, the mathematics and science activities are built around 'concepts in common', with the principle that the learning in each subject enriches learning in the other. For instance, measuring, graphical work and data modelling generally are freshly developed in science contexts in ways that raise questions and promote deeper knowledge in science, and the science context raises questions that can be further explored mathematically.

Stages of the IMS Pedagogical Model

Orienting: Teachers pose questions, explore students' ideas and orient them to the learning focus by a variety of means such as asking for predictions, questioning what they have noticed, asking for ideas about what could be measured, and eliciting prior knowledge. This provides a way to focus students' attention on what is worth noticing about the school environment, or about data sets for instance, and could be interesting to explore.

Posing representational challenges: Students are challenged to explore and represent their ideas and practices, for instance they may be challenged to represent the movement of their shadow over a day, involving decisions about what to measure and how to represent patterns in length, and angle, or to use particle representations to predict, investigate and explain why a saucer of water evaporates more quickly in warm, or windy places.

Building consensus: This involves two stages. First, using the student ideas and representations to compare, evaluate and then synthesise these to reach agreement about which aspects of these effectively show patterns in data, or suggest explanations. Second, these ideas are refined by students, and consolidated to establish a shared understanding of the concept and associated representations. In this process students develop knowledge of the role of representational work in learning.

Applying and extending conceptual understanding: Students are given new representational challenges to extend their new knowledge and practices in related situations, or further concepts are introduced through representational tasks, to repeat the cycle.

In these stages the teacher is constantly monitoring and responding to students' representations and ideas. The approach can be seen as 'assessment as learning'. The focus on student production has been found to allow the teacher significant insights into student thinking. The art of teaching in this way involves setting appropriate tasks, preparing students strategically through questioning and challenges, and guiding their work to reach consensus about the key ideas and their representations. The sequences all involve a close association of material exploration, and the generation of ideas.

These stages have much in common with the 5Es that underpin Primary Connections (PC). The stages line up as Orienting = Engage, Posing Representational Challenges = Explore, Building Consensus = Explain; and Applying and Extending Conceptual Understanding = Elaborate. The 'Evaluate' stage appears in the IMS pedagogy as a continuous process of monitoring and formative assessment (assessment 'for' and 'as' learning) throughout the stages. Most sequences have a summative evaluative task, but this sits outside the cycle. Distinct from the 5Es, the IMS stages are explicitly focused on representations as central to learning (consistent with the PC focus on literacy) and structured to lead from noticing what is of interest to investigate, through the generation of representations, to generating class agreement on key concepts as systems of representations and representational practices.

The teaching and learning sequences follow these stages explicitly, but they cycle in different ways, in different lessons and in different topics. In some lessons there are more than one cycle, or even interweaving cycles for science and mathematics. In other cases, a cycle is spread over a number of lessons. Sometimes, activities have more than one role, such as an extension representational challenge acting as an orientation into a further concept. Nevertheless, we believe the movement from opening up what is noticed, to exploration and representation construction, to evaluating and building consensus, is a fundamental and powerful aspect of effective teaching and learning. Tasks in the sequences are designed to be approachable at a range of levels. This, together with teacher open questioning and targeted scaffolding, enables differentiation of the learning.

Supporting differentiation of learning in the IMS learning design

In the IMS learning sequences the student- guided inquiry design enables diverse student learning needs to be responded to within the regular classroom. The open learning tasks are designed flexibly to enable students to work at their own level, and at their own pace, to develop their understanding and skills in a variety of ways. Variation in student responses offers a resource for promoting, encouraging and refining learning as students demonstrate, in different ways, what they know and understand. With teacher support, students learn from each other's ideas and productions. The focus on student-constructed representations, and open questioning and discussion, enables the teacher to monitor individual students' understandings and cater for their learning needs over time.

Features of the learning sequences that enable embedded and teacher-supported differentiation

There are three distinct aspects of the IMS pedagogy that enable differentiation.

Open questioning, guided inquiry and open tasks provide the teacher with insight into individual student learning and understanding that:

a) enables teacher decisions for on-the-spot feedback, and individualised monitoring and support of student learning through targeted learning adjustments, scaffolding, and extension challenges.

> *"Giving them (students) more freedom is a good approach because they're more capable than I thought they would be, but they still needed the support as well. So, giving students the initial freedom to do whatever they thought they could do and then helping them from that…"*

b) enables support for students to navigate tasks with multiple entry points, solution pathways and outcome possibilities, whilst negating possible student stigmatisation from the withdrawal from their peer group, or students assigned a different task.

> *"the fact that they are open-ended so they (the students) can come to a solution in a variety of different ways. There was not one student where I had to really modify an activity for, they could participate in the activity, they could all have success in the activity but they all got something from it and because it was open-ended…"*

c) enables the development of creative and critical thinking skills, and higher-order thinking, as student responses are not limited

> *"…I always found everything was just deeper level thinking."*

Peer learning, collaborative learning and student voice increases student engagement as students learn from and with their peer group.

Students learn collaboratively as a whole class and in mixed ability peer groups. Student are encouraged to share ideas, co-construct investigations, designs, data and representations. Through purposeful guided reflection, targeted scaffolding, prompts and extension challenges, students engage in comparative discussions and review of peer representations (e.g. graphical representations) to build their understandings.

> *"…we were able to cater for everyone without making it obvious to them that we had to modify the activities, which I think is really important for their confidence and self-esteem and learning too."*
>
> *"…coming from their peers and it's quite interesting because when they actually get feedback from their peers as well I find that they really do put it into practice a lot quicker, it's quite interesting, as opposed to coming from the teacher all the time, it's coming from someone different. That has been a really interesting pick up that we have found..."*

Multimodal representational challenges cater for diverse learner needs and provide differentiated insight into students' conceptions.

Teachers have identified that a focus on multimodal representation enhances learning for students with language difficulties, who are English Second Language (ESL), and/or have literacy support needs, since they are not so constrained by their language skills. Access to multiple modes reduces the effects of language demands as barriers to learning. Students' multimodal representations provide teachers with insight into individual students' knowledge, skills and learning needs.

> *"…this has been really interesting, seeing children that don't speak up as often really come up with some really insightful representations. I mean, they're a lot further ahead than what I thought."*
>
> *"show me what you know through your drawings' and often that speaks volumes because children find it difficult to articulate at the time. They might understand more than what they are conveying… But they are actually showing me so much of their knowledge through their diagrams."*

Investigating Our Heights - Body Maths (Year 5): Sequence Overview

In this **teaching and learning sequence** students will focus on the measurement of length/height in a familiar context and ways of representing data. Students estimate and order their heights using formal units (m/cm), develop understanding of variability, interpret ideas of range, mean, and mode and organise and represent data(categorical/cluster/frequency data). They apply concepts to new data sets.

Lesson Sequence - Outline

Lesson 1: Estimating student's individual heights and constructing, interpreting and recording class display (pre sequence assessment task)

Teachers determine students understandings about measurement with the pre sequence assessment task. Why do you need to know your height? (Establish partner students in Kindergarten for investigation) e.g., Could all students in our class go on theme park/water park rides requiring the minimum height of 1.4m?

Lesson 2: Measuring heights and constructing and interpreting a class display.

Measuring (using student choice of tools) individual student's heights to nearest cm, construct and represent class display and compare with previous class display of estimates (lesson 1); Interpret and produce data display as collection of individual measures. Making informal inferences

Lesson 3: Inventing new ways of representing and interpreting class data.

Use class height data and display to invent new ways of representing the data e.g., line plot/dot plot/bar graph. Discuss relative effectiveness of data displays for interpretation of data Predicting heights for other classes of same grade and/or partner students and representing the predicted data as a class display or other form of representation.

Lesson 4: Using our data to predict and measure heights by grade level and age.

Collect real height data from one or more different grades or partner students.

Lesson 5: Devise methods for organizing and displaying the combined data as a combined grade display (post sequence assessment task)

Interpret data for ideas of range, mode and average height for each grade or partners and represent changes with increasing age of group. Students then complete written post sequence assessment task exploring interpretation of graphical representations, and understanding of metric units m and cm, average, mode and range.

Curriculum Focus: Science and Mathematics Learning

Learning Focus	Key Curriculum Outcomes (Victorian Curriculum)
Science ideas and practices • Link ideas about height to human growth; do we grow taller at a constant rate? Why? Why not? • Are there gender differences in the way we grow taller at particular stages of development? • Living things grow with observable features (height). • Collecting data and constructing, representing and interpreting data display. • Making informal inferences by reasoning about distribution and variation.	**Science** **Biological sciences** Living things grow, change and have offspring similar to themselves (VCSSU043) **Recording and processing** Use formal measurements in the collection and recording of observations (VCSIS068) Construct and use a range of representations, including tables and graphs, to record, represent and describe observations, patterns or relationships in data (VCSIS085) **Analysing and evaluating** Compare data with predictions and use as evidence in developing explanations (VCSIS086)
Mathematics ideas and practices • Estimating, measuring and representing variability (height). • Select and use appropriate formal measures of length and convert between m and cm including use of decimal notation. • Collecting data and constructing, representing and interpreting data display. • Developing skills in constructing graphical representations. • Developing ideas of average, range and mode. • Making informal inferences by reasoning about distribution and variation.	**Mathematics** **Using units of measurement** Choose appropriate units of measurement for length (VCMMG195) **Fractions and decimals** Compare, order and represent decimals (VCMNA190) **Data representation and interpretation** Pose questions and collect categorical or numerical data by observation or survey (VCMSP205) Construct displays, including column graphs, dot plots and tables, appropriate for data type, with and without the use of digital technologies (VCMSP206) Describe and interpret different data sets in context (VCMSP207)

Investigating our Height - Body Maths: Equipment/Resources

Lesson		Equipment/Resources
All Lessons		**Students:** student workbooks (unlined), pencils, colours and rulers, 1cm and 2cm grid paper **Teachers:** Board (IWB/whiteboard), and or butchers' paper for shared recording and pens Sticky notes (5-7cm squares) Height chart or tape measure (vertical) Paper tape String Review Questions response sheet
1	Estimating heights and constructing and interpreting class display	Pre Sequence Assessment Task (as above)
2	Measuring heights and constructing and interpreting a class display	Completed height sticky notes from lesson 1 and all lessons resources
3	Inventing new ways of representing and interpreting class	(as per all lessons)
4	Using our data to predict and measure heights by grade level (age)	Complete height data tables and all lesson resources
5	How do we grow taller over time across the grades?	(as per all lessons) Post Sequence Assessment Task

LESSON 1 – Estimating heights and constructing and interpreting class display

(Approximate duration 60 minutes)

Learning focus:

Science ideas and practices

- Formal measures can be used to estimate observable features of humans
- Living things grow and change

Mathematics ideas and practices

- Estimating and ordering their heights using formal units (m/cm)
- Understanding variability
- Interpreting ideas of range, mean, mean and mode
- Organising and representing data (categorical/cluster/frequency data)

Learning intention:

- Students will estimate their own height and represent the data as a whole class display to interpret the range and average.
- Students will explore methods to construct a class display and practise recording the class display in preparation for lesson 2.

Equipment/Resources

Equipment required for all lessons

Students: student workbooks (unlined), pencils, colours and rulers, 1cm and 2cm grid paper

Teachers: Board (IWB/whiteboard), and or butchers' paper for shared recording and pens
Sticky notes (5-7cm squares)
Height chart or tape measure (vertical)
Paper tape
String
Review Questions response sheet

The lesson at a glance:

In this lesson students estimate student's individual heights. The predict whether students will be more or less than the maximum height of 1.4m to gain entry to a ride. They collect the data and co-construct, interpret and record it as a class display.

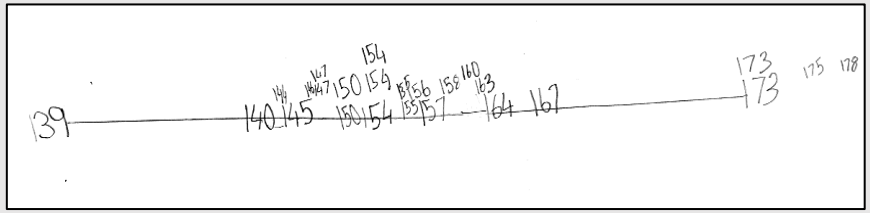
LESSON 1 – Estimating heights and constructing and interpreting class display

(Approximate duration 60 minutes)

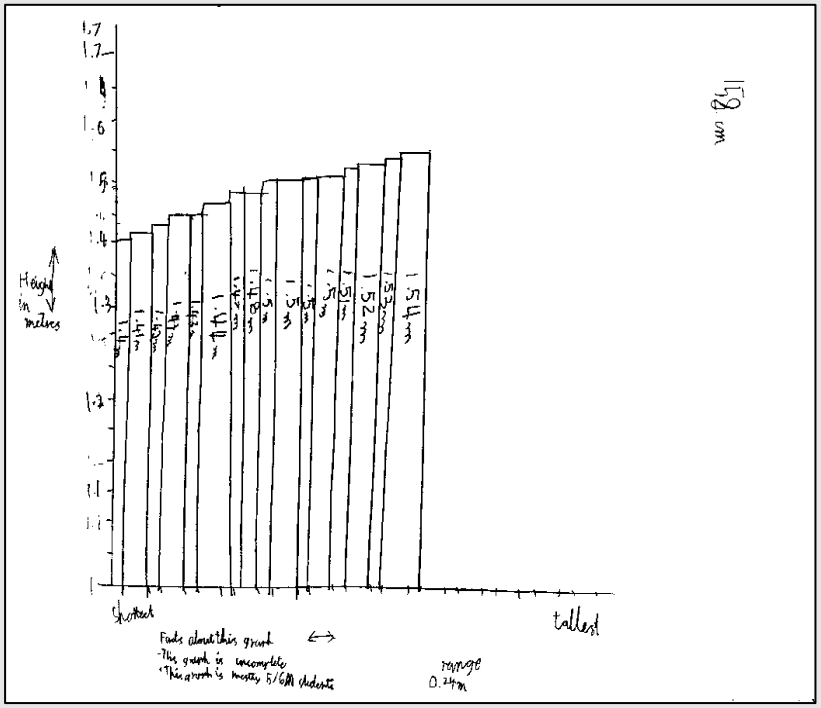
Learning focus	Pedagogical stage	Lesson Outline *(NB: time allocations a guide only)*	Monitoring and supporting learning
Mathematics: Estimating and ordering their heights using formal units (m/cm) Understanding variability Interpreting ideas of range, mean and mode **Science:** Living things grow and change Formal measures can be used to estimate observable features of humans	***Orienting*** Students estimate individual height. Height is related to growth and age	**Whole Class Discussion** *(10 minutes)* Establish purposes of knowing own heights ❖ Why do you need to know your height? **Probing Questions** ❖ *Could all students in our class go on theme park/water park rides requiring the minimum height of 1.4m?* ❖ *Do you think there will be a typical height of students in our class? Or many different heights?* ❖ *When do you think school-aged students grow the fastest? In what grade or grades?* ❖ *What methods would be best to record, display and compare all student predictions?*	Do students make reasonable estimates of the height of 1.4? Do they make reasonable estimates of the heights of students in their grade using formal units? What informal notions of rate of growth do they demonstrate?
Mathematics: Estimation Measuring and recording Understanding variability Interpreting ideas of range, mean and mode		**Discussing the investigation; establishing formal measures** *(15 minutes)* Discuss how we notice similarities and differences about our students' heights. ❖ *Do you think there will be a typical (mode) height? Or many different heights?* Class activity is to make a line of students standing in order of height and then ask students to make groups of same/similar heights. ❖ *What can we say about our class heights without even estimating or measuring the heights?* ❖ *What groups or clusters of measures do we have?* **Individual task** Students estimate own height. Students record independently their estimates on sticky notes. NB. Initial estimates can be inferred from observations of height of shortest, middle and tallest three students.	Can students recognise that there may be a typical height? Can students recognise the order of height of individuals, and in groups to show same height? (clusters)

Mathematics: Organising and representing data (categorical/cluster/frequency data)	***Posing representational challenges*** Students think about how to collect and organise data and construct interpret and represent a class display	**Whole Class: Representing the data** *(15 minutes)* Teacher places sticky notes randomly on board in order to construct a class display. Class discussion about how to organise the sticky notes. Ask students to think about making a display on the board/chart that shows other people the measurements at a glance. Explain that the display should help other people quickly see anything they think is important or worth noticing, such as a trend (a tendency, a pattern) or relationship among the measurements. **Students in pairs** Students (2 or more selected students) **invent** ways to organise the sticky notes in some order or clusters	How do students organise clusters or groups of similar measures? How effective and accurate are students' representations of the data?
Mathematics: Organising and representing data (categorical/cluster/frequency data) Compare data with predictions and use as evidence in developing explanation Make statements about comparisons	***Building consensus*** Identifying similarities and differences ***Building consensus*** consolidating, refining representations	**Whole Class Data Display Review** *(15 minutes)* Students **evaluate** and discuss what does our display show well? Teacher sums up the range of measures to get students thinking about 'the range and the average' **Individual task** Students record their observations about the class display (simple statements and represent the class display as a graph in their work book). Select a few students' drawn representations that show a range of features and use these for class discussion. NB: A sketch of the class display as a draft would be a good scaffold before making a more accurate attempt. Students are encouraged to comment on the range and shape of the data display. NB. These statements could be recorded on the board or on separate strips of paper that can be easily attached to the class graph **Teacher Notes** Students may need to **refine** their representations. Drawing the class display may take students several attempts and scaffolding by teacher (potential maths time continuation) Some students may need to use sticky notes to replicate the class display on a large piece of paper before recording in their workbooks. The use of 2cm grid paper rather 1cm grid might be helpful for some students	Can students explain what the class display shows in different ways? What are students' notions of average and range? How reasonable are students explanations and statements about the display?

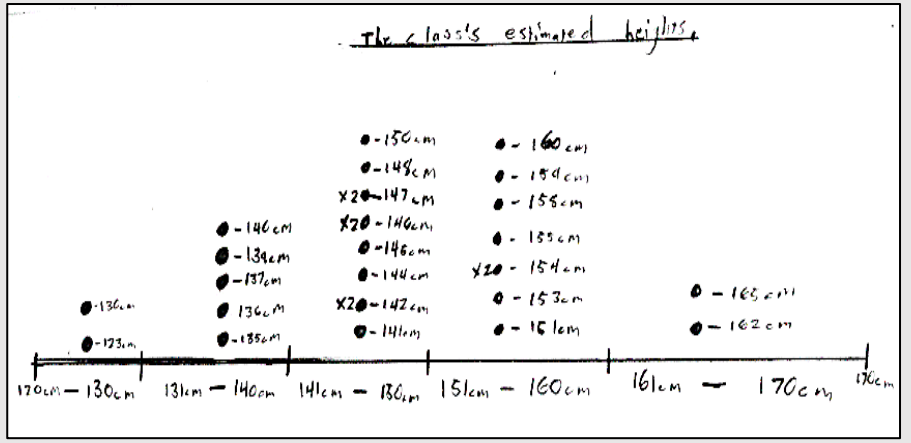
Samples of student work



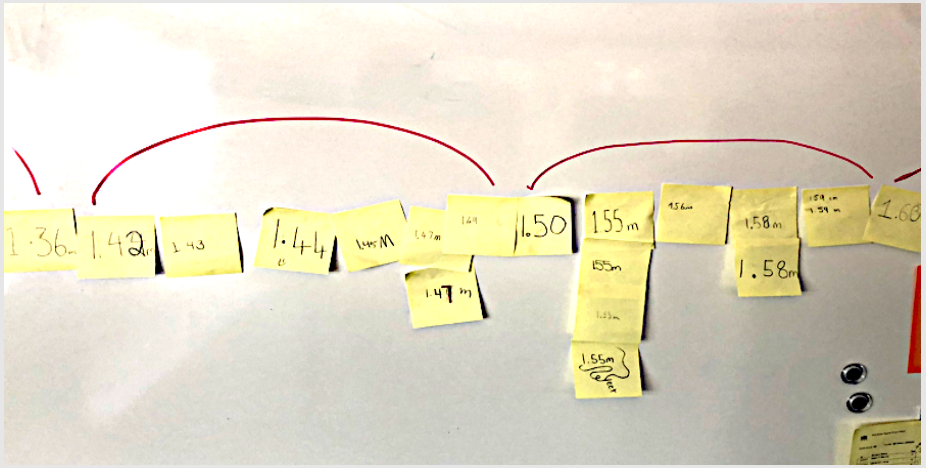
Estimated student heights represented on horizontal line plot.



Variation in response heights represented as column graph in ascending order



Student heights represented as clusters on horizontal line plot with intervals shown in 10 cm segments.



Whole class display of individual student ordering of individual predictions in order from shortest to tallest.

Mathematics: Identify differences between representations and their meanings Estimate proportions (not tall enough) Draw inferences Plan data collection to answer questions	***Building consensus*** Identifying similarities and differences	**Gallery Walk** *(10 minutes)* Students compare and contrast others' representations and ideas. ❖ *What can you tell from the different representations?* ❖ *How effective are they?* ❖ *What do they show?* ❖ *What don't they show?* **Review Questions (whole class discussion and individual response)** ❖ *How accurate do you think your estimate was?* ❖ *Why do you need to be accurate?* ❖ *What proportion of students in your own grade level did you estimate would be less than 1.4m i.e., not tall enough to go on the ride yet?* ❖ *How did you work this out? (Link to work on percentages and decimals)* ❖ *What would you need to do to collect data to answer this question?*	How do students' responses to review questions provide evidence of their thinking and interpretation of the data?
Mathematics: Apply knowledge of range and mode to other samples	***Applying conceptual understanding and representational challenges***	Students **extend** their learning about range and average to apply to new data exploration investigations such as estimating height of students from other classes of the same grade level and classes from other grade levels.	Can students extrapolate their experience to come up with other investigations that involve discussion of measurement processes, sample size, appropriate comparisons?

LESSON 2 – Measuring heights and constructing and interpreting a class display

(Approximate duration 90 minutes)

Learning focus:

Science ideas and practices

- Formal measures can be used to estimate observable features of humans
- Construct and use a range of representations, including tables and graphs, to record, represent and describe observations, patterns or relationships in data

Mathematics ideas and practices

- Estimating and measuring their heights using formal units (m/cm)
- Height is variable

Learning intention:

- Students will measure (using student choice of tools) individual student's heights to nearest cm, construct a new class display and compare with the class display of estimates (lesson 1)
- Students will estimate a fixed measure – e.g., (1.4m ride height)

The lesson at a glance:

In this lesson students measure their individual heights. They collect the data and co-construct, interpret and record it as a class display. They give reasons for choosing most effective methods of data display

Equipment/Resources

Completed sticky notes from lesson 1
(to reference)

Equipment required for all lessons

Students: student workbooks (unlined), pencils, colours and rulers, 1cm and 2cm grid paper

Teachers: Board (IWB/whiteboard), and or butchers' paper for shared recording and pens
Sticky notes (5-7cm squares)
Height chart or tape measure (vertical)
Paper tape
String
Review Questions response sheet

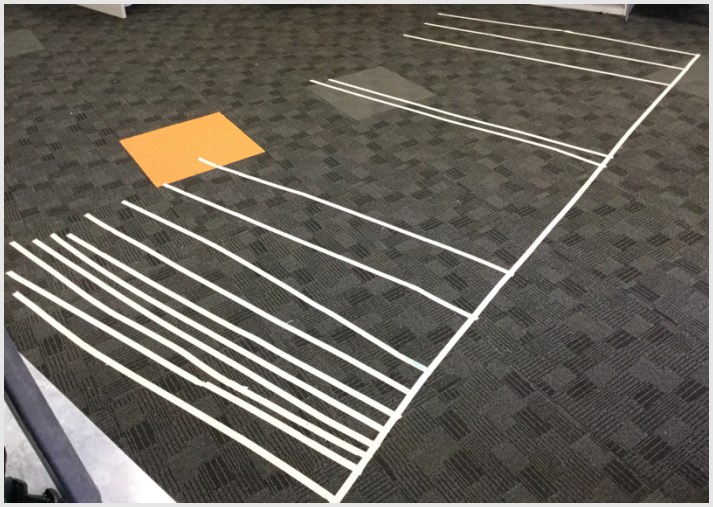
LESSON 2 – Measuring heights and constructing and interpreting a class display

(Approximate duration 90 minutes)

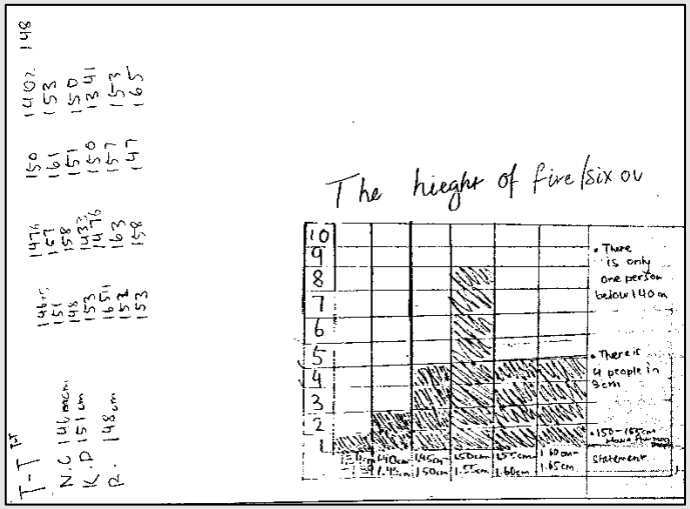
Learning focus	Pedagogical stage	Lesson Outline *(NB: time allocations a guide only)*	Monitoring and supporting learning
Mathematics: Estimate and measure their heights using formal units (m/cm) Height is variable.	***Orienting*** Students estimate and measure individual height. Reason about whether there may be a typical height for our class(mode).	**Introduction (whole class)** *(5 minutes)* **Probing Questions** ❖ *Can we find out if our students (in this class) would be Ok to meet the 1.4m restriction for ride entry?* ❖ *Can we estimate where a 1.4 m height would be? (How may centimetres would this be?)* Teacher selects student/s to place a marker on the board or wall as students' estimate.	Can students make reasonable estimates of a 1.4m height and convert to centimetres easily?
Science: Construct and use a range of representations, including tables and graphs, to record, represent and describe observations, patterns or relationships in data	***Posing representational challenges*** Students think about how to collect and organise data and construct interpret and represent a class display	**Whole Class Discussion** *(10 minutes)* ❖ *In what ways can we measure our height?* ❖ *What tools or materials can we use?* ❖ *Does it need to be accurate?* Teacher may also suggest use of paper tape or string or using a marker. ❖ Do we need a formal measure such as a metre rule or a tape measure or height chart? Why? Why not?	Are students' methods for measuring appropriate and feasible?

Mathematics: Use practical tools and strategies to measure heights Understand the need for a standard unit of measure **Science:** Measurement processes. Variation in height, and growth patterns with age	***Posing representational challenges*** Students think about how to collect and organise data and construct interpret and represent a class display	**Height investigation in pairs** *(20 minutes)* In pairs students use paper tape to measure partner's height. Students may decide to lay on floor or stand next to wall. Discuss the exact point of measure (base of foot to top of head). Students record their name on back of tape. **(pairs of students)** **Data display considerations: Whole class discussion** ❖ What do we want our display to show? Ask students to think about making a display on the board/chart that shows other people the measurements at a glance. The teacher explains that the display should help other people quickly see anything they think is important or worth noticing, such as the range of heights. **Class Display of recorded (tape lengths) student heights** *(15 minutes)* Students place their tape on chart/board (at baseline) to form a class display ❖ Why does the tape need to be placed in alignment with others? Teacher may need to highlight or draw baseline **Data Co-construction** Students **invent** ways to organise the tapes (heights) in some order or clusters. ❖ What can we do to order the heights so we can find out what is typical? ❖ Do we have heights that are identical or that are very close, or clumped together? (This is the mode). ❖ Do we have some that are very different? (outliers)	Do students make accurate measures of the heights of students in their grade? Can they use formal units of measure accurately (m, cm)? Can students identify data displays that bring out key features? What understandings or misconceptions of "average", "range" and "mode" have they developed?
Mathematics: Understanding variability Interpreting ideas of range, mean, mean and mode	***Building consensus*** Identifying similarities and differences	**Data Display Analysis: Group or Whole Class** *(5 minutes)* Students **evaluate** and discuss what does our display show well? Teacher sums up the range of measures to get students thinking about the range, the mode and the mean 'average'.	How have students' developed inferential reasoning about distribution and variation from their graphs?

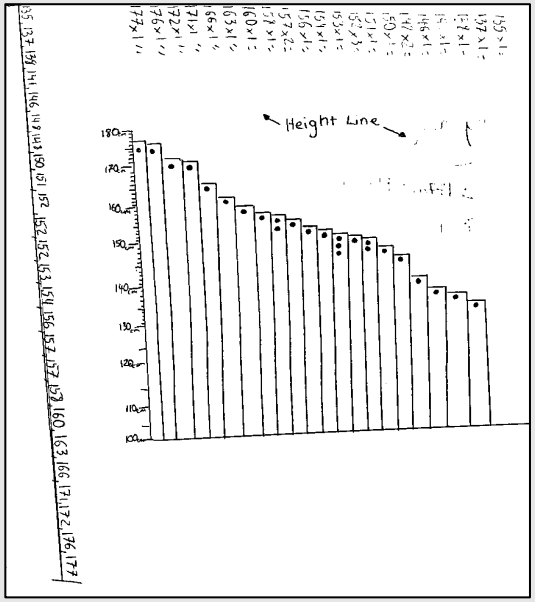
Samples of student work: Some samples of students' measurement records (tape) on chart/board (at baseline) to form a class display
Students **invented** ways of organising and representing data in some order or clusters.
NB: How do these examples show student understanding of baseline and the range of heights?



Students attempt at ordering height (whole class measures)



Student representation of class heights in a column/bar graph



Column graph of heights using a scale in intervals of 10cm with dots highlighting the heights

Mathematics: Interpreting and comparing data	***Building consensus*** Identifying similarities and differences	**Measuring and representation** *(10 -15 minutes)* **(groups or individual tasks)** As for Lesson 1, students record their observations about the class display Compose simple statements and represent the class display as a graph in their work book or use graph paper. Students are encouraged to comment on the range of measures and the shape of the graph they have recorded in their workbook. Students at this grade level should be using formal units of length easily and showing relationship between m and cm. **Teacher Notes:** A sketch of the class display as a draft would be a good scaffold before making a more accurate attempt.	How effective and accurate are students' representations of the data? Can students explain what the class display shows in different ways? How do the examples show student understanding of baseline and the range of heights?

Mathematics: Interpreting and comparing data display Using and representing proportions	***Building consensus*** consolidating and refining representations	**Review questions** *(10 minutes)* **(whole class discussion and individual response)** How accurate do you think your estimate was? Why do you need to be accurate? • Students review measuring metres as centimetres and making conversions between measures i.e., 1.4m =140cm. • Students can be encouraged to sketch the display freehand and may need to **refine** their representations. Students draw the class display in their work books, which may take students several attempts **(Individual task)** NB. Some students may need to use graph paper from the outset. Students compare their real class data with the estimated heights class display. What do you notice that is similar or different? What proportion of students in **our grade level** did you estimate would be less than 1.4m i.e., not tall enough to go on the ride yet? How does the height data that we have collected and represented match or differ from our estimates in lesson 1?	How reasonable are students explanations and statements about the display? How do students' responses to review questions provide evidence of their thinking about the need to measure accurately? Are students able to estimate or calculate the proportion of students less than 1.4m using fractions or percentages?
Mathematics: Interpreting and analysing data sets and representations	***Building consensus:*** Consolidating and refining repesentations	**Gallery Walk** *(10 minutes)* Students compare and contrast others' representations and ideas. ❖ *What can you tell from the different representations?* ❖ *How effective are they?* ❖ *What do they show* ❖ *What don't they show?*	

LESSON 3 – Inventing new ways of representing and interpreting class data

(Approximate duration 60 minutes)

Learning focus:

Science ideas and practices

- Construct and use a range of representations, including tables and graphs, to record, represent and describe observations, patterns or relationships in data

Mathematics ideas and practices

- Represent different ways to represent a class display. Construct and interpret dot or line plots or other graphical forms
- Develop understanding of range and mode

Learning intention:

- Students will develop understanding of the aggregate nature of data when creating displays and apply scale to the representation of data

The lesson at a glance:

In this lesson students think about different ways to represent a class display. Students copy the table of class height data into their workbooks and students construct dot or line plot to represent the data

Equipment/Resources

Equipment required for all lessons

Students: student workbooks (unlined), pencils, colours and rulers, 1cm and 2cm grid paper

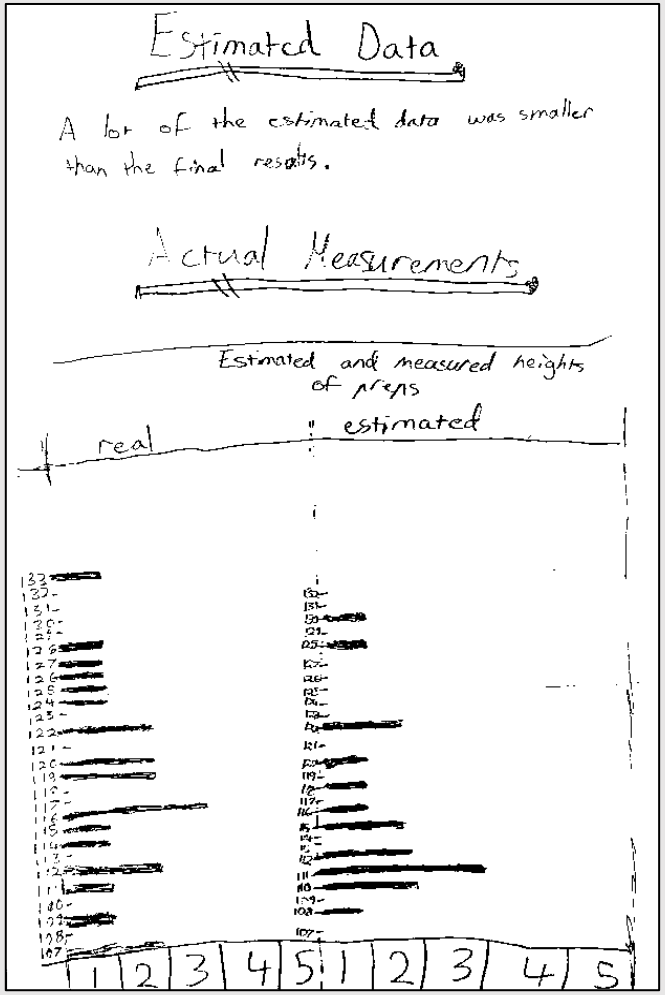
Teachers: Board (IWB/whiteboard), and or butchers' paper for shared recording and pens
Sticky notes (5-7cm squares)
Height chart or tape measure (vertical)
Paper tape
String
Review Questions response sheet

LESSON 3 – Inventing new ways of representing and interpreting class data

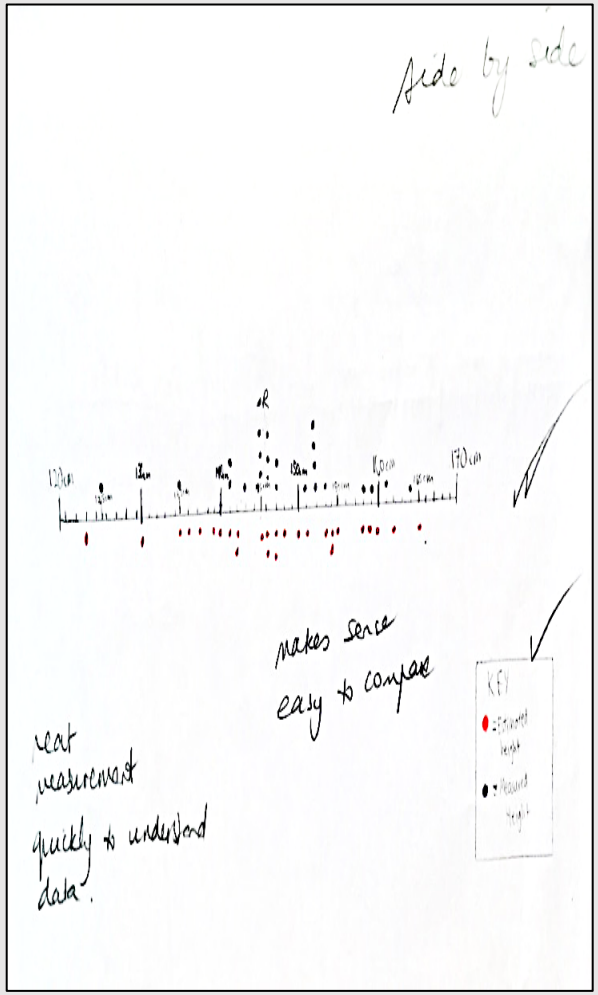
(Approximate duration 60 minutes)

Learning focus	**Pedagogical stage**	***Lesson Outline*** *(NB: time allocations a guide only)*	**Monitoring and supporting learning**
Mathematics: Inventing different ways of representing data	***Orientation and posing representational challenges*** Students think about different ways to represent a class display	**Introduction (whole class)** *(10 minutes)* **Posing Questions:** *Can we invent a new way/s to represent our class display?* *Could we show the heights of our students using different types of graphs?* Students come up with some examples. Teacher may refer to an example of dot plot or line plot. *What have these representations shown?*	Do students suggest suitable forms of displaying and graphing the data? How do these examples demonstrate students' understanding and skills in choosing and using representations? (dot/line plots)
Mathematics: Representing data as line or dot plots; horizontal bar graphs **Science:** Construct and use a range of representations, including tables and graphs, to record, represent and describe observations, patterns or relationships in data	***Posing representational challenges*** looking for relationships between data and the representations	**Discussion: Class display** *(20-minutes)* Teacher directs students to the class display of heights. Students are selected to complete a data table on board showing heights and frequencies as a class display. **(Individual students)** ❖ *How can we show our class data differently so that it is easy to see what the most common height/s are?* Teacher constructs dot/line plot to show heights and frequencies. What was the most common height? Students copy the table of class height data into their work books (for easy reference). Students construct dot or line plot to represent the data. (**Groups or individual)**	How effective are students' attempts to draw dot/line plot or other display?

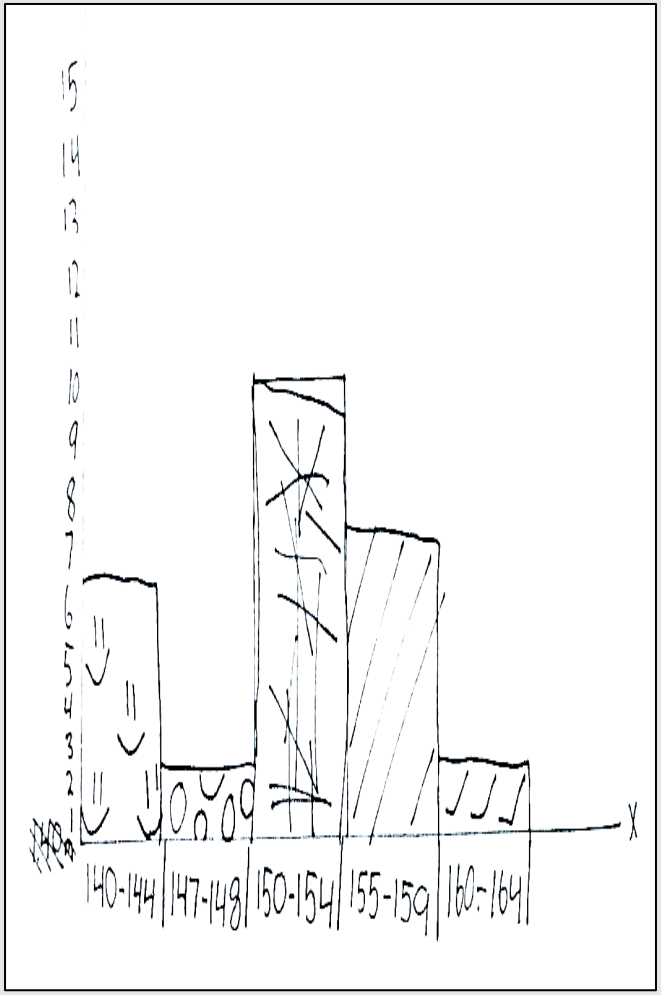
Samples of student work: Different representations of class height including sstudent drawings of dot or line plots to represent the data



Horizontal bar graphs of estimated and actuals heights



Estimated (below the horizontal scale) and actual heights as dot plot with teacher feedback

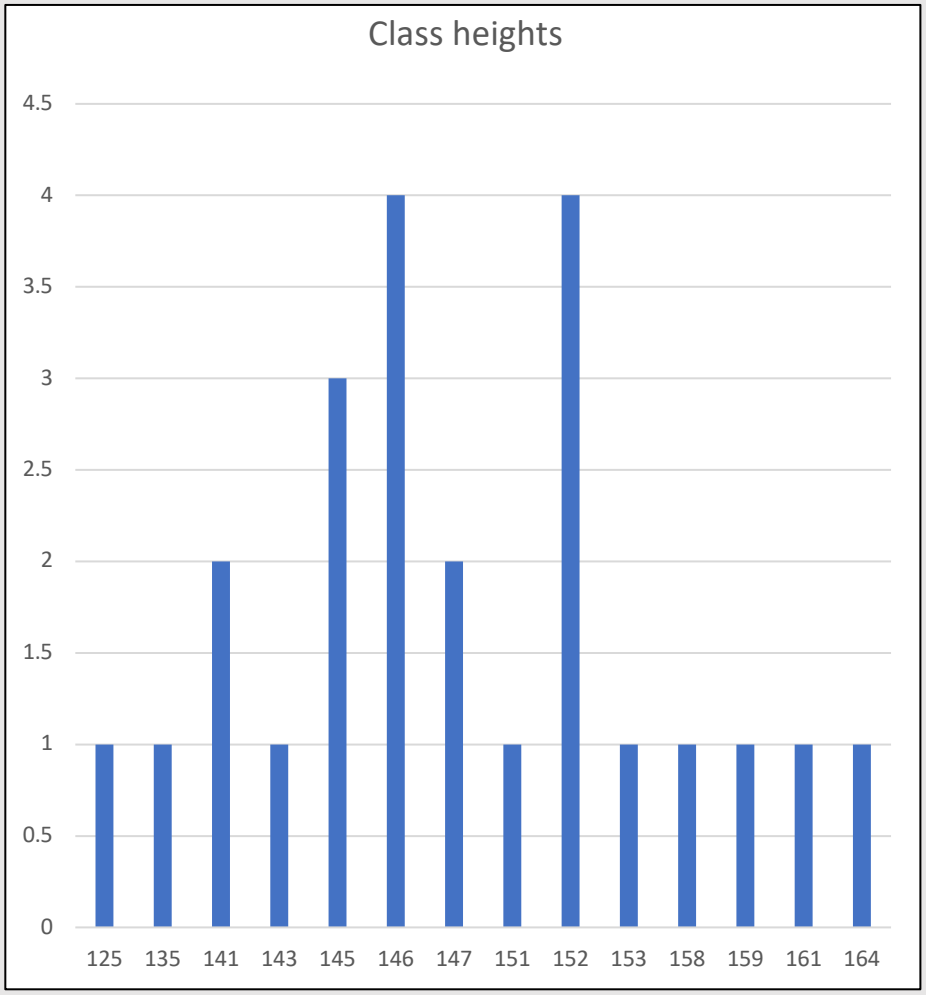


Student representation of clusters (binned) of heights. The labelling of the two axis.

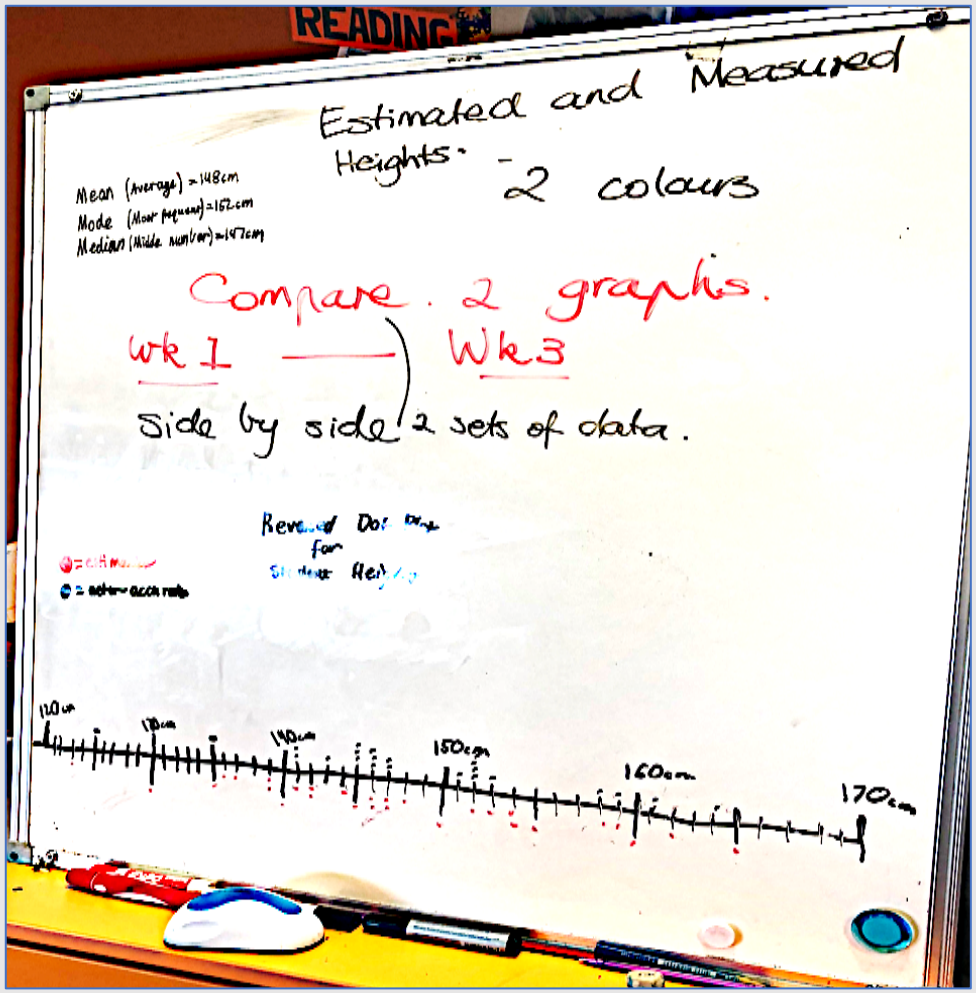
Mathematics: Interpreting variety of representations	***Building consensus*** synthesizing data for agreement and pattern identification. Students consolidating and refining representations	**Whole Class Discussion** *(5 minutes)* Teacher selects several different examples for discussion. ❖ *How is this plot/s useful for seeing the range and typical heights of students?* ❖ *Is this form of display easier to interpret than the class display?* Students **evaluate** and discuss what does our class display shows well? **Representations as graphs** *(20 minutes)* **Individual task** As for Lesson 2, students sketch the line/dot plot as a draft before making a more accurate attempt. Students can be encouraged to sketch the display freehand and may need to **refine** their representations. Students are encouraged to comment on the shape of the display they have recorded in their workbook. **Teacher Notes:** The teacher may need to review constructing a scale (to show frequencies) and how to coordinate the data on a vertical or horizonal axis. Students may need to be guided to notice that their dot or line plot shows clusters of same heights. **Whole class discussion and individual response** *(10 minutes)* ❖ How easy is it to interpret your new display (line/dot plot)? ❖ In what ways are your previous recordings of the class data modified to enhance your new representation. ❖ What do you notice that is similar or different?	In what ways do students provide evaluative comments about the effectiveness and suitability of each display How well do students interpret and explain what their graphs show? Can they justify why they choose this form of graph? How reasonable are students explanations and statements about the display? How do students' responses to review questions provide evidence of their thinking and review of their representations?
	Building consensus consolidating and refining representations	**Gallery Walk** *(10 minutes)* **Whole class discussion and individual response** Students compare and contrast others' representations and ideas. ❖ *What can you tell from the different representations?* ❖ *How effective are they?* ❖ *What do they show?* ❖ *What don't they show?* Teacher focuses on the range of measures to get students thinking about the relative use of dot/line plots or other graphs.	In what ways do students compare and contrast the effectiveness and suitability of different representations? How do the samples show the representation of the data easily

	Applying conceptual understanding and representational challenges	**Extending the data set** Student-led activity where teacher records student **estimates** of typical (average) height by grade level i.e., Prep- Grade 6) by constructing a table of data as a class display. "Estimates of typical height for each grade"	

Samples of class board work – Sharing and consensus examples



Whole class derived bar graph of class heights from excel spreadsheet.



Student derived representation of estimated and actual heights presented as dot plot in clusters

LESSON 4 – Using our data to predict and measure heights by grade level (age)

(Approximate duration 60 minutes)

Learning focus:

Science ideas and practices

- Humans grow and change over time
- Estimating growth of body in height over time (ages and grades)

Mathematics ideas and practices

- Estimating and ordering height in m and cm
- Devising sampling strategies and understand concept of sampling
- Collect data in effective way

Learning intention:

- Students will apply their learning from interpreting prior data to predict then measure student heights for other classes of same grade or partner student and represent the data as a class display (or other form of representation).
- Students will collect a sample from other grades and focus on sampling strategies

Equipment/Resources

Equipment required for all lessons
Students: student workbooks (unlined), pencils, colours and rulers, 1cm and 2cm grid paper

Teachers: Board (IWB/whiteboard), and or butchers' paper for shared recording and pens
Sticky notes (5-7cm squares)
Height chart or tape measure (vertical)
Paper tape
String
Review Questions response sheet

The lesson at a glance:

In this lesson students will apply their learning from lessons 1-3 to think about the variability of the heights of other grades or whole school data. They collect data suitable to their context and make inferences about the heights of students of other ages.

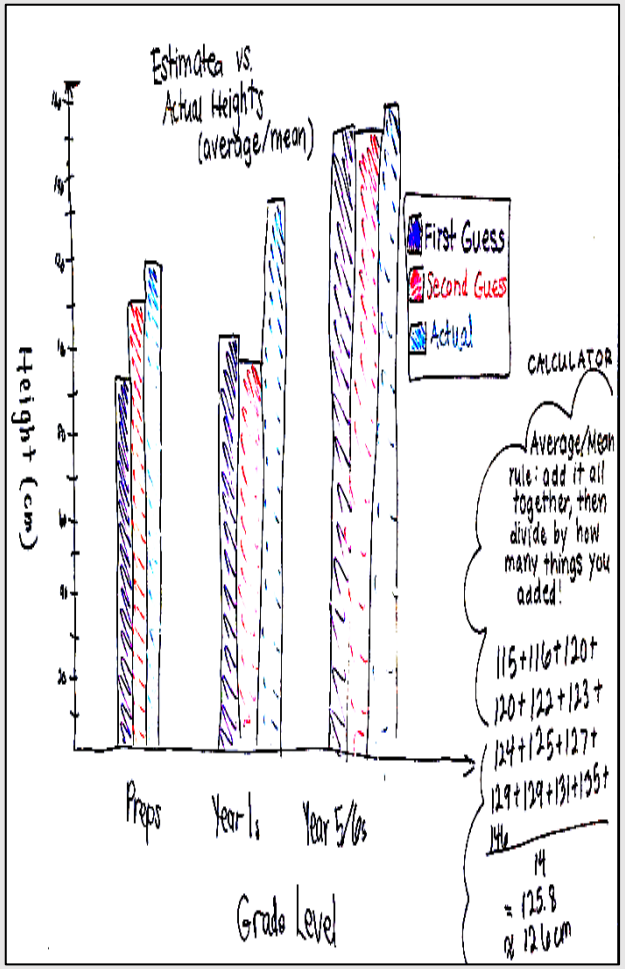
LESSON 4 – Using our data to predict and measure heights by grade level (age)

(Approximate duration 60 minutes)

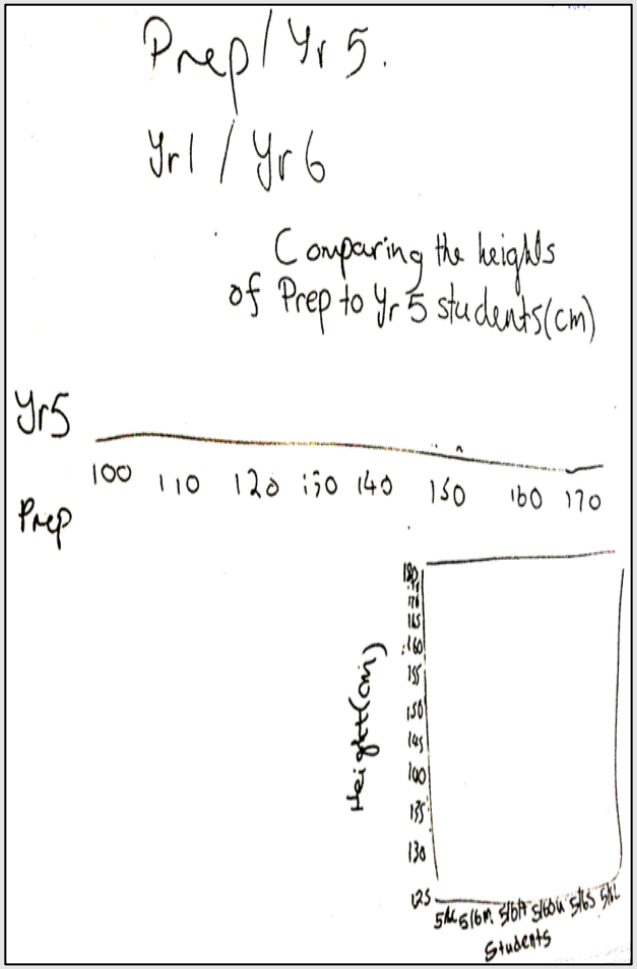
Learning focus	Pedagogical stage	Lesson Outline *(NB: time allocations a guide only)*	Monitoring and supporting learning
Science: Estimating growth of body in height over time (ages and grades)	***Orienting*** Students think about different ways to sample the heights of students across the school	**Introduction (whole class)** *(10 minutes)* **Posing Questions** ❖ *Can we find out if students in other grades would be Ok to meet the 1.4m restriction for ride entry?* ❖ *How would we go about this if you couldn't measure the height of every student in the school?* ❖ *What do you estimate would be the typical height for students in each grade level or your partner student?*	What are students ideas about sampling? What do students' understand about the idea of typical height? How reasonable are their estimates of typical height by grade level?
Mathematics: Estimating and ordering height in m and cm Devising sampling strategies	**Posing representational challenges** looking for relationships between individual class data and a sample of school heights	**Group Investigation: Measurement** *(20-30 minutes)* Students organise to measure actual heights of selected classes or partner students, other classes at same grade level or other grades to measure (in cm) and record (in a table) NB. Alternatively students could collect a sample of height measurements from other grades. How would they select students to represent the range of heights?	How well are students able to measure and construct a two-way table of data?
Mathematics: Constructing a two-way table Purpose of sampling	***Posing representational challenges*** Students think about how to compare and represent data sets	**Recording and representing data group or individual tasks** *(20 minutes)* Students record the table of data showing estimates by grade and partner students and the actual table of height data in their workbook. Use the table of data of actual heights of students from other classes/grades Student make simple statement about what their table of data shows. Students are encouraged to comment on the range of measures and the shape of a graph that might represent these combined data. NB. Some students may need review or assistance in constructing a two-way table and making accurate entries of the height measures.	How well are students able to construct and interpret a two-way table of data? How do students identify and interpret features of the data sets overall?

Mathematics: Describe or calculate differences between estimates and actual data	***Building consensus*** Find similarities and differences between estimates and actual data	**Review questions** *(10 minutes)* **Whole class discussion and individual response** Students compare the combined table data with their own class data. What do you notice that is similar or different? ❖ *What is the range of height measures from the larger sample?* ❖ *What proportion of students in **the combined group data** are less than 1.4m i.e., not tall enough to go on the ride yet?* ❖ *How does the height data that we have collected and represented match or differ from our estimates?* ❖ *Can you explain how you would take a sample of heights for each class.*	Can they organise the larger data set appropriately? (If used) What sampling strategies did they apply? Why? How do students' responses to review questions provide evidence of their thinking and review of their estimates?
Mathematics: Making inferences from the data. Identifying range and any common measures	**Building consensus** *Assessing* and synthesizing students' ideas and representations to establish the affordances of different representations (e.g. dot plot)	**Gallery Walk** *(10 minutes)* Students compare and contrast others' representations and ideas. ❖ *What can you tell from the different representations?* ❖ *How effective are they?* ❖ *What do they show?* ❖ *What don't they show?* Teacher focuses on the range of measures to get students thinking about the differences in the combined data set. Teacher Note: See examples of student work attached	Ask students to describe the range of heights from the combined data and anything special they notice about the data.

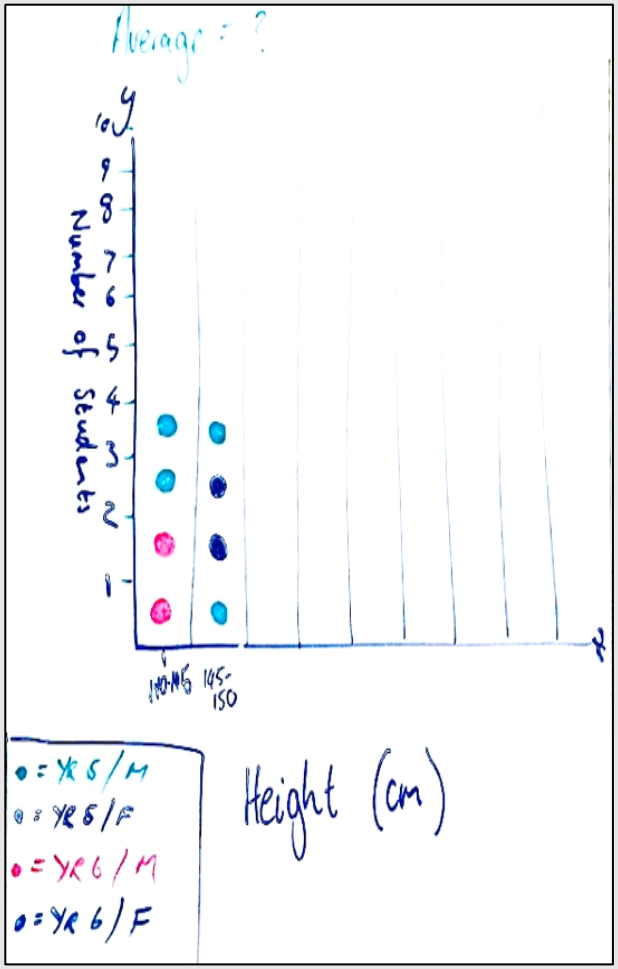
Samples of student work: Examples of co-constructed class display and tables of data
How do the examples show students understanding of the features of the combined year level data?



Whole class co-constructed data employing a side by side column graph to compare the estimated heights and 'actual' recorded heights



Whole class board: Students representing ideas for different ways of presenting the data



Incomplete whole class co-constructed data representation using colour coding for frequency and binning

LESSON 5 – How do we grow taller over time across the grades?

(Approximate duration 90 minutes + Post Sequence Assessment Task)

Learning focus:

Science concepts and representational processes:

- Humans grow and change over time
- Represent growth of body in height over time (ages and grades)

Mathematics concepts and representational processes:

- Reason about their sampling strategies
- Invent ways to represent combined measures as a graph
- Use a range of representations, including tables and graphs, to record, represent and describe observations, patterns or relationships in data

Learning Intention:

- Students will interpret table of combined data to invent a combined class display
- Students will develop understanding that there are similar values or clumps in the data

Equipment/Resources

*Table of data from Lesson 4
Post Sequence Assessment Task

Equipment required for all lessons
Students: student workbooks (unlined), pencils, colours and rulers, 1cm and 2cm grid paper

Teachers: Board (IWB/whiteboard), and or butchers' paper for shared recording and pens
Sticky notes (5-7cm squares)
Height chart or tape measure (vertical)
Paper tape
String
Review Questions response sheet

The lesson at a glance:

In this lesson students will represent and explain the variability of the heights of other grades or whole school data. They will make reasonable graphs of the data and explain features such as range and mode. The idea of rate of change can be explored

Lesson Preparation

Prior to this lesson students height data entered into a table of data during lesson 4 must be completed and accurate.

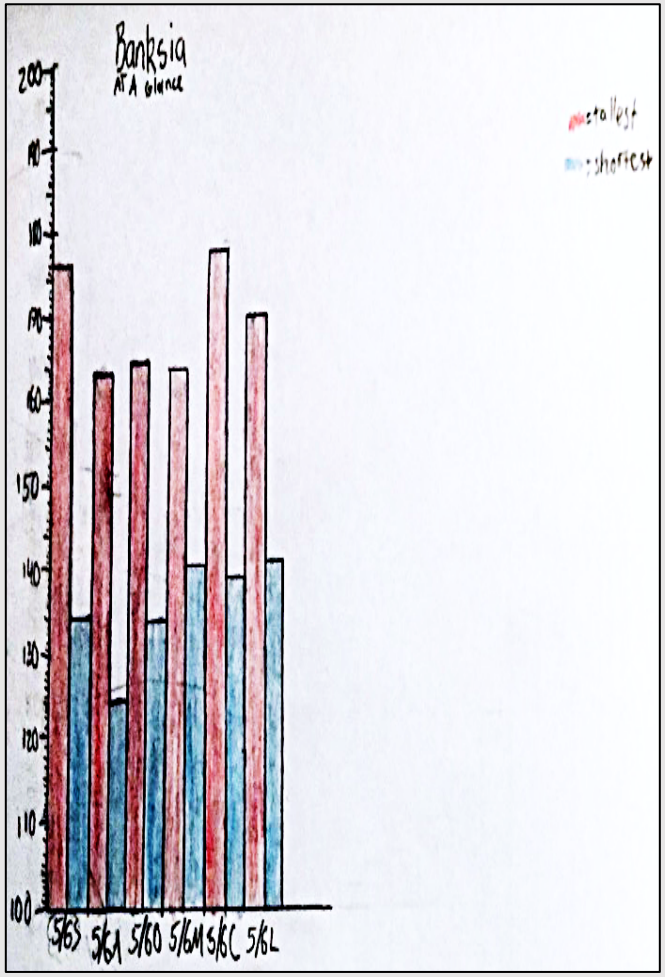
LESSON 5 – How do we grow taller over time across the grades?

(Approximate duration 90 minutes + Post Sequence Assessment Task)

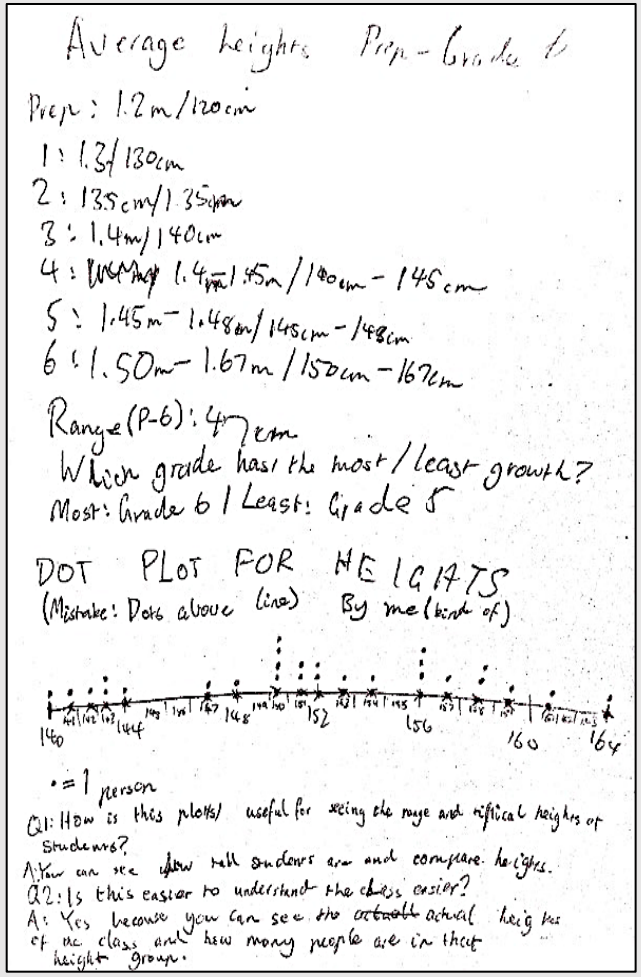
Learning focus	Pedagogical stage	Lesson Outline *(NB: time allocations a guide only)*	Monitoring and supporting learning
Science: Representing growth of body in height over time (ages and grades) **Mathematics:** Invent ways to represent combined measures as a graph	***Orienting*** Students make connections between the data and their estimates of growth rate	**Whole Class Discussion** *(5 minutes)* **Posing Questions:** Revisit initial questions posed: ❖ *When do you think school-aged students grow the fastest?* ❖ *In what grade or grades? or is it a steady growth?* ❖ *How does your data help you answer this question?* ❖ *When do you think most students in the school would reach the 1.4m height to allow entry to the water/theme park rides?*	Do students focus on the amount of growth from one grade to the next or attempt to see a rate of growth?
Interpreting data to answer and pose new questions Apply understanding of proportions	***Applying conceptual understanding and representational challenges*** Focus on observing and describing patterns of growth in each class and for the overall data set	**Group discussion or individual** *(5 minutes)* Encourage students to pose new questions and investigations e.g., is there a difference in the pattern of height growth for boys and girls? Could you work out the connection between uniform size or shoe size according to height? Why is this information useful?	How appropriate are the new questions posed by students for further investigation as class display? Can students explain relationships or differences between features of the data?
Science/Maths: Use a range of representations, including tables and graphs, to record, represent and describe observations, patterns or relationships in data	***Building consensus*** Teacher supports joint construction representing an effective data display	**Whole Class Discussion and Representation** *(20-30 minutes)* Students refer to lesson 4 table of data "Heights of students from grade x to x" Students co-construct combined table of data with teacher as a class display. Students (2 or more selected students) **invent** ways to organise the combined data into one combined group display as a graph. ❖ *How can we indicate grade differences? What type of graph should we use to best represent these data?* ❖ *Do we have heights that are identical or that are very close, or clumped together? Do we have some that are very different? (outliers)*	How well do students build on prior experience of representing class data? What features are noticeable in their data representations that are improved or more explicit?

Samples of student work: Students' data representations demonstrating range

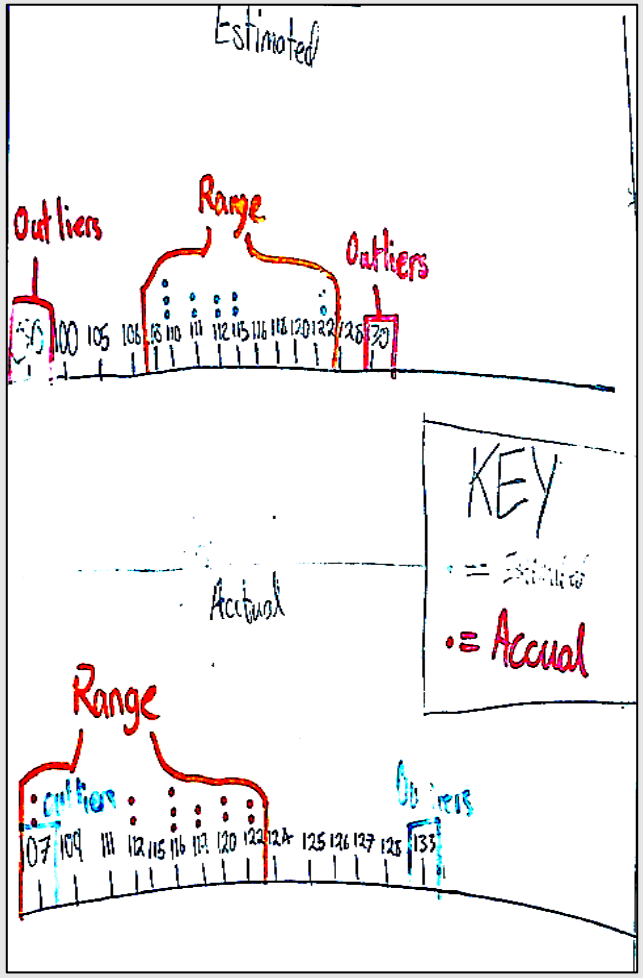
How do the samples allow students to observe patterns in the data and interpret the range and the mode?



Student representation of range of heights, 'tallest' and 'shortest', in each class.



Student data recording and dot plot showing typical height



Individual student dot plots showing range

Mathematics: Ideas of range, mode and average Develop idea of a growing pattern Make inferences about the combined data and the effectiveness of the data display Develop ideas about sampling	***Building consensus*** Evaluating and reaching agreement on effective data displays and its interpretation Question students to stablish reasons for identifying mean and mode and affordances for each	**Whole Class Height Data: Review** *(15 minutes)* Students **evaluate** and discuss what our display shows well? Teacher sums up the range of measures to get students thinking about the range, the mode and the mean 'average' of the combined data. **Individual's representations of Whole Class Data** Students record the combined table of height data, record observations and represent the display as a graph in their work book**.** A sketch of the class display as a draft would be a good scaffold before making a more accurate attempt. Students are encouraged to comment on the range of measures and the shape of the graph they have recorded in their workbook. **Teacher Notes:** The teacher may need to review the methods to show scale and frequencies, and intervals of time (grades) in a graph. Drawing the combined grades' display may take students several attempts and scaffolding by teacher (potential maths time continuation) Some students may need to use graph paper from the outset. **Whole Class - Review questions** *(10 minutes)* ❖ *Do these data show you when students grow the most? How?* ❖ *What proportion of students in **the combined group** are less than 1.4m i.e., not tall enough to go on the ride yet?* ❖ *How did you work this out from the table of combined data or the graph?* ❖ *How does the height data that we have collected and represented match or differ from our estimates?* ❖ *What can we say about how height growth in our school?* ❖ *Is a sample of heights from each grade or just some grades going to give you reliable data to answer your questions?*	Students records questions and their responses in work book. What do students 'choices about their graphical representations reveal about their thinking? What do students notice easily from the graph? What features do they overlook? Why? Ask students to describe the range, and mode (what's the most common height) from combined height data. (If used) What sampling strategies did students suggest? Why? Do students recognise the importance of accuracy in height measures?

		Gallery Walk *(10 minutes)* Students compare and contrast others' representations and ideas. ❖ *What can you tell from the different representations?* ❖ *How effective are they?* ❖ *What do they show?* ❖ *What don't they show?*	How well have students developed in their choices of appropriate representations? What features of the displays do they use to justify their choices?
Investigate relationships between proportions of body parts and their measures	***Applying and extending conceptual understanding and representational challenges*** Extending students' ideas and further problems to apply knowledge of relationships between measurements	**Further investigations** *(10 minutes)* **Whole class discussion and individual response** Encourage students to pose new questions and investigations e.g., is there a difference in the pattern of height growth for boys and girls? ❖ Could you work out the connection between uniform size or shoe size according to height? Why is this information useful? ❖ What other body measurements could be related to your height? e.g., your arm span? or length of feet and height? ❖ What are some other interesting body measurements could we investigate e.g., proportion of neck to waist measurement?	Can students use the insights from previous investigations to suggest coherent approaches to measurement and data generation and what analyses may be carried out?
		Post sequence assessment task *(15-20 minutes)* Written tasks for all students. Teacher explains task with students answering independently. Answers can be shown through text or drawings.	

Samples of Year 5 student responses: Post Sequence Assessment Task

Example 1

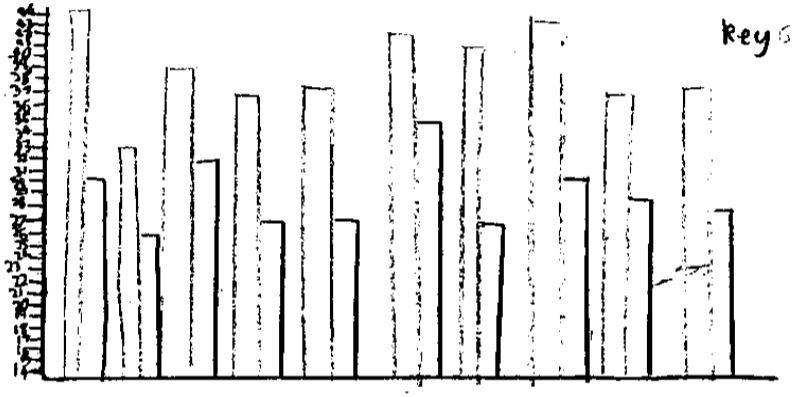
How fast do our feet grow?

The grade 6 class at Errol Parade school were trying to remember how small their feet had been in Grade 1. They found out the shoe size of their Grade 1 buddies to compare with their own shoe sizes using large shoe size numbers like 30 (European shoe sizes). There were 11 students in each class and the sizes were:

Grade 1 30 26 32 27 27 35 29 27 30 29 28

Grade 6 44 33 39 37 38 43 39 41 44 37 37

1. Construct a representation or draw a graph of these numbers that makes it easy to compare the Grade 6 and Grade 1 students' shoe sizes.



Use your representation to answer the following questions

2. What is the median value of the Grade 6 class shoe sizes?

44

3. What is the mode of the Grade 6 class shoe sizes?

37

4. Which of these – the median, or mode, do you think is a better measure of the typical shoe size? Explain why.

Mode because most people would have the same size.

5. How much bigger is the 'typical' shoe size for Grade 6 students, compared to Grade 1? Show how you decided that.

37 and 27 these numbers are the numbers in the middle.

6. Is there more, or less variation in shoe size for the Grade 6, compared to Grade 1? Explain how you decided that.

Less because Grade 1 has 8 shoe sizes and Grade 6 has only 7.

Sample of Year 5 student responses: Post Sequence Assessment Task

Example 2

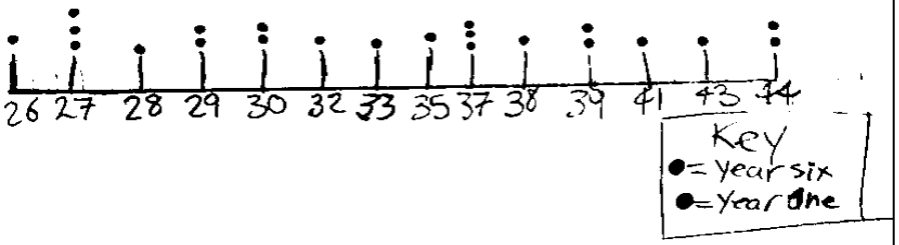
How fast do our feet grow?

The grade 6 class at Errol Parade school were trying to remember how small their feet had been in Grade 1. They found out the shoe size of their Grade 1 buddies to compare with their own shoe sizes using large shoe size numbers like 30 (European shoe sizes). There were 11 students in each class and the sizes were:

Grade 1 30 26 32 27 27 35 29 27 30 29 28

Grade 6 44 33 39 37 38 43 39 41 44 37 37

1. Construct a representation or draw a graph of these numbers that makes it easy to compare the Grade 6 and Grade 1 students' shoe sizes.



Use your representation to answer the following questions

2. What is the median value of the Grade 6 class shoe sizes?

39

3. What is the mode of the Grade 6 class shoe sizes?

33

4. Which of these – the median, or mode, do you think is a better measure of the typical shoe size? Explain why.

median because it tells you that there is a middle.

5. How much bigger is the 'typical' shoe size for Grade 6 students, compared to Grade 1? Show how you decided that.

10 numbers inbetween lots of the numbers.

6. Is there more, or less variation in shoe size for the Grade 6, compared to Grade 1? Explain how you decided that.

I thought of the age differences because year 6s have had more time to grow.

Sample of Year 5 student response: Post Sequence Assessment Task

Example 3

Name:_____

How fast do our feet grow?

The grade 6 class at Errol Parade school were trying to remember how small their feet had been in Grade 1. They found out the shoe size of their Grade 1 buddies to compare with their own shoe sizes using large shoe size numbers like 30 (European shoe sizes). There were 11 students in each class and the sizes were:

Grade 1 30 26 32 27 27 35 29 37 30 29 28

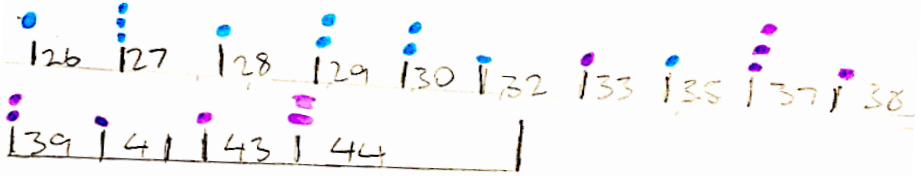
Grade 6 41 33 39 37 38 43 39 41 44 37 37

1. Construct a representation or draw a graph of these numbers that makes it easy to compare the Grade 6 and Grade 1 students' shoe sizes.

Key

Blue = grade 1

Purple = grade 6



Use your representation to answer the following questions

2. What is the median value of the Grade 6 class shoe sizes?

39

3. What is the mode of the Grade 6 class shoe sizes?

37

4. Which of these – the median, or mode, do you think is a better measure of the typical shoe size? Explain why.

median because you know where the whole year level is around or they get an idea of where these class members are to.

5. How much bigger is the 'typical' shoe size for Grade 6 students, compared to Grade 1? Show how you decided that.

8 because the median of grade 1 is 131 and the median of grade 6 is 139.

6. Is there more, or less variation in shoe size for the Grade 6, compared to Grade 1? Explain how you decided that.

There is the same amount of range between the numbers and the same amount of numbers that were in the shoe [illegible] have the same amount of variation because of when one grade has a really small foot compared to others and a grade 1 has a foot that is bigger than others compared to his classmates

Post Sequence Assessment Task

Name:_____

The grade 6 class at Errol Parade school were trying to remember how small their feet had been in Grade 1. They found out the shoe size of their Grade 1 buddies to compare with their own shoe sizes using large shoe size numbers like 30. There were 11 students in each class and the sizes were:

Grade 1	30	26	32	27	27	35	29	27	30	29	28
Grade 6	44	33	39	37	38	43	39	41	44	37	37

1. Construct a representation or draw a graph of these numbers that makes it easy to compare the Grade 6 and Grade 1 students' shoe sizes.

Use your representation to answer the following questions

2. What is the median value of the Grade 6 class shoe sizes?

3. What is the mode of the Grade 6 class shoe sizes?

4. Which of these – the median, or mode, do you think is a better measure of the typical shoe size?

5. How much bigger is the 'typical' shoe size for Grade 6 students, compared to Grade 1? Show how you decided that.

6. Is there more, or less variation in shoe size for the Grade 6, compared to Grade 1? Explain how you decided that.