Connecting mathematics and science through an Interdisciplinary Mathematics and Science (IMS) project in the primary school

Joanne Mulligan
Macquarie University, NSW
<joanne.mulligan@mq.edu.au>

Melinda Kirk
Deakin University, Melb.Vic
<melinda.kirk@deakin.edu.au>

Russell Tytler
Deakin University, Melb.Vic
<russell.tytler@deakin.edu.au>

Peta White
Deakin University, Melb.Vic
<peta.white@deakin.edu.au>

In this article the authors provide an overview of the Interdisciplinary Mathematics and Science project conducted in Grades 1 to 6 in four Australian schools. The project teachers implemented an enquiry-based pedagogical model through 12 interrelated topics. This paper details how a range of mathematics and science concepts can be effectively taught through an interdisciplinary approach based on representational invention.

The Interdisciplinary Mathematics and Science (IMS) project aimed to explore the learning possibilities generated by posing questions and solving problems that connect mathematics and science learning (see https://imslearning.org), across a range of grade levels. A key challenge was to generate tasks and learning sequences where science investigations productively engaged students in mathematics learning and vice versa.

We built upon the classroom-based work that showed synergies across mathematics and science in data, statistics, measure and space (Lehrer & Schauble, 2012; Mulligan, 2015). The approach has generated deeper student mathematical and science knowledge beyond teacher and curricular expectations. We were also aware of the possible difficulties in achieving authentic interrelated learning between mathematics and science (Lehrer & Schauble, 2020), and our research has aimed to develop approaches to resolve such difficulties.

Developing the interdisciplinary mathematics and science learning sequences

In developing learning sequences reflecting the IMS approach we drew on the Australian Curriculum aims and scope to align concepts and processes where integration provided an advantage for the learning in each (ACARA, 2018) with some lessons adapting Primary Connections (Australian Science Academy, 2020). Concepts and processes common to both mathematics and science include: the development of data representations and statistical reasoning including concepts such as variability, sampling, and measures of central tendency. Measurement concepts and units, and spatial concepts of mapping, area, angle and rotation as well as number concepts such as calculation, using decimals and ratio and proportion were embedded. In selecting topics, we found that the complementarity of concepts could occur in different ways. In a unit on changes to materials, cooking activities provided shorter-term opportunities for developing ideas about measure and fractions. In most topics, the focus shifted back

and forth between mathematics and science across different time scales and with different degrees of formalisation, with each repeat of the learning sequence resulting in more sophisticated conceptual growth and representations and refinement of understandings.

Table 1 describes the way that mathematics concepts have been intertwined with the science concepts in learning sequences. In all but one, the sequence is identified by the science context, but the mathematics is developed strategically and synergistically alongside the science exploration, and feeds back into further scientific representational work. For each sequence, students are challenged and supported to invent, evaluate and refine mathematical representations in response to a science-based question about measure, data representation or spatial or numerical patterns. Students are thus introduced to underlying mathematical ideas which are progressively refined through a guided inquiry process.

Table 1. Interrelationships between science and mathematics concepts in learning sequences (Adapted from Tytler et al., in press). Details of some of these sequences can be found at https://imslearning.org/resources/.

Grade	Topic	Science	Mathematics
1, 4	Astronomy	Astronomy: Shadows, sun movement, explaining day and night: modelling, earth and space perspectives.	Angle as rotation and length of shadow, formal and informal measure, graphing shadow length, time sequencing, perspective taking.
1, 4	Ecology	Ecology: Living things, diversity, distribution and adaptive features related to habitat.	Data modelling of living things in sample plots, tables and graphs, space and mapping, measurement, area, coordinates, directionality, sampling, using a scale.
1, 4	Motion	Representing speed, distance and time relations, constant speed, acceleration.	Embodied representation of relation between distance, time and speed, length measures, modelling variation, graphing speed, distance, time, distance-speed-time relations for motion down a ramp.
2	Chemistry	Dissolving and mixing, physical change, particle ideas, chemical reactions, change to materials in cooking.	Representing time sequences of mixing and dissolving under different conditions, timing cooking- measurement for cooking recipes: standard and informal units, fractions, proportion.
2, 4	Fast plant growth	Plant growth, structure and function, growth needs and patterns, plant life cycles including germination, flower structures, fertilisation process.	Measurement of plant height, leaf size, shape and number, tracking growth over time, tables, line graphs, units (cm and mm- formal units), time intervals, using a scale.
2	Force and motion: Helicopters	Force: Flight and air flow, gravity, representing forces, modelling and design, variable control.	Measure of helicopter parameters, time, data variation, data modelling, number line.
2	Water use	Personal use and conservation of water.	Estimation and measure of water use, informal and formal measures of volume.
3	Heat and temperature	Heat sources and heat flow, temperature measurement, conduction, insulation and design.	Constructing a scale; relation between informal and formal measures, timing and sequencing trials, variation and representing data.
4, 5, 6	Light	Properties of light: Vision, reflection and image creation, refraction.	Angle, rotation, reflection, directionality, symmetry.
5, 6	Measurement: Body height	Height measure and variation, differences between populations	Measurement (m and cm), data modelling, variation, measures of central tendency and of variation, comparison through graphs, categorising/organising data, sampling.
5, 6	Astronomy/ space and the solar system	Solar system, day and night, planetary features, moon movement and phases.	Conceptualising ratio of planetary size and distance, angle measurements for moon observations, compass points, tracking position over time, perspective taking from earth and space, representing cosmological distances: powers of ten.
6	Magic micro-organisms	Structure and function of micro-organisms; bacteria, fungi. Investigative skills.	Multiplication, ratio, area, measurement.

In each of the learning sequences (except the Measurement: Body Height sequence), the science context has driven mathematical inquiry by creating a need to explore and represent underlying patterns (spatial, numerical) in ways that feed back into questions that drive science representational work. In some sequences this is an iterative process that involves more than one cycle of stages focused on the refinement of the same concept (e.g., variability and graphical work) or on a sequence of concepts (shadow patterns involving length and angle variation related to the sun's movement, leading into modelling earth rotation.

The IMS pedagogical approach

Our pedagogical approach was based on posing meaningful, open-ended problems, challenging students to construct/invent representations and with teacher guidance critically negotiate refinement of these to scaffold students towards science and mathematics understandings and practices. The approach requires of teachers the ability to meaningfully represent concepts and the deeper meanings of mathematical practices. The project involved scaffolding the learning sequences with the teachers (see https://imslearning.org) by following a cycle of processes shown in Figure 1.

Orienting

This stage entails challenging and supporting students to notice key, productive features of the situation, and establishing a need for exploration through constructing representations in both science and mathematics. The teacher explores student ideas to orient the class toward a productive framing of engagement with the phenomenon, and potential methods of inquiry drawing on mathematical representational work.

Posing representational challenges

Students are challenged, individually or in groups, to invent/construct representations that reflect a process of claim-making and predictive and causal reasoning or justification. The process involves students in meaningful exploration, measurement and data modelling, feeding into their representations.

Building consensus

This stage entails teacher guided sharing/display, comparison/evaluation and feedback on the clarity and explanatory power of the representations. The teacher guides refinement/revision, drawing strategically on the variation in students' representations to guide an emerging consensus.
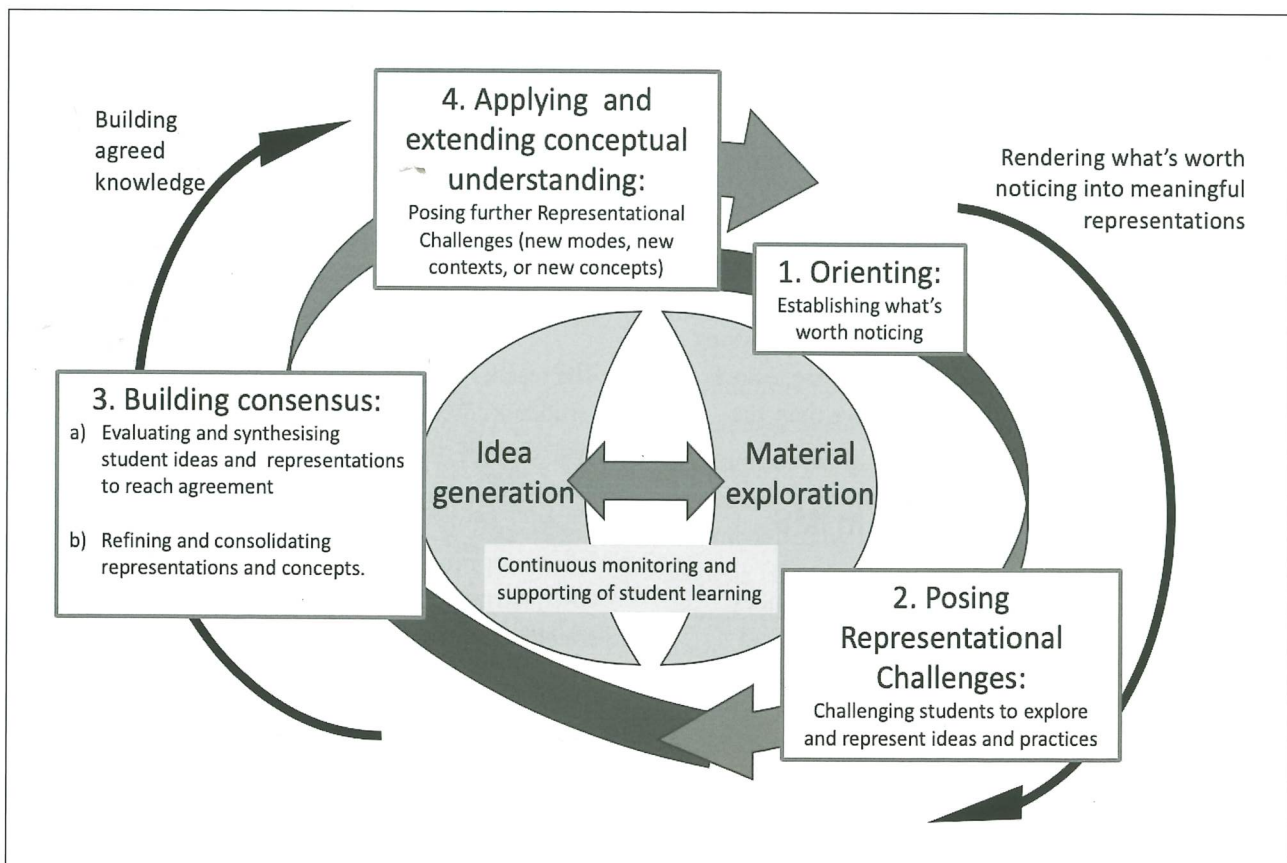

Figure 1. IMS Learning pedagogical model.

Applying/extending understandings to new settings

In this stage students respond to new and related challenges. This may involve coordinating different representations in solving new problems or refining/applying representations to related challenges.

Implementing the project in primary schools

The research team worked collaboratively with four different schools—two metropolitan and two regional—involving 35 teachers over three years and six cohorts of students from Grades 1 to 6. Each grade level cohort involved one or more classes of students so that some grades provided 75 students (three classes) and others one class (25 students). Two representative groups of students were tracked for three years; one group from Grade 1 through to Grade 3 and the other group from Grade 4 through to Grade 6. This was an important part of the project; monitoring the learning of the same students over this time but involving a new set of teachers each year to collaboratively generate the learning sequences with the research team.

At the commencement of each year professional learning workshops were provided to participating or newly recruited teachers, reflective workshops conducted at the end of the year, and planning and review meetings conducted during the topic implementation. The learning sequences were planned by the researchers and refined in consultation with the classroom teachers through these workshops and meetings. Detailed lesson guidelines and associated resources were provided to teachers for each learning sequence, with examples of student work highlighted from comparable cohorts. Learning sequences were implemented in each of three or four school terms, ranging from between three and eight weekly lessons of one or two hours duration. Usually, these lessons involved the science learning time rather than the mathematics time slot.

Examples of student learning in IMS

We illustrate, through several learning sequences, examples of the ways in which mathematics and science were interrelated, describing the pedagogical approach, students' representations of their learning, and student and teacher interview commentary.

Ecology in the school yard: Grades 1 and 4

In a learning sequence on living things in the school yard Grades 1 and 4 students engaged in mapping and data representations of organisms found in individual plots, which raised questions of particular animal distribution across habitats. This led to further data modelling of the distribution of organisms across plots, and subsequent discussion about adaptation to habitats. The mathematics involved student generation and refinement, through sharing and teacher guidance, of tallies of living things in the sample plots, tables, and finally graphs to provide a visual account of the distribution. Figure 2 is a Grade 1 student's graphic representation of the number of spiders across the different plots. The student explained at interview: "…the numbers were different (pointing to plot numbers) because there were lots of plots in different areas for different habitats, for different animals".
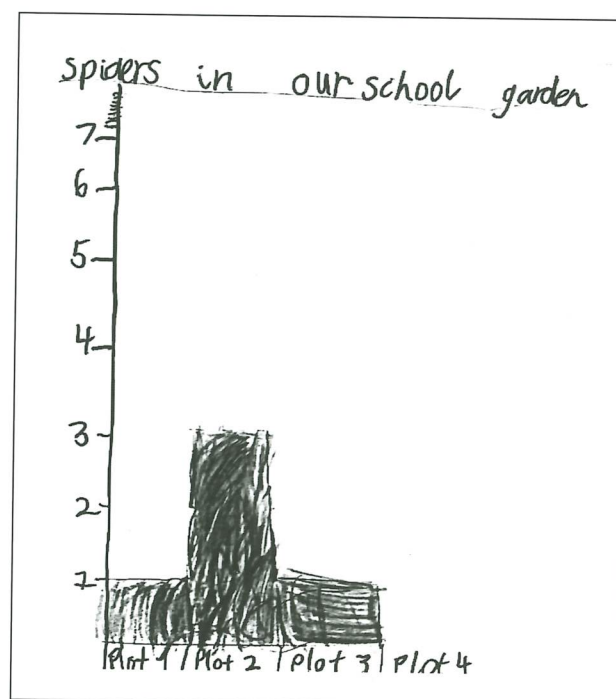

Figure 2. Grade 1 student's bar graph representation of the number of spiders in different plots.

The teacher of a Grade 1 class commented on how the students contributed to the building of appropriate graphs through their ideas and directions, with the mathematical purpose driving the representational work.

It was not modelled by me but a modelled, co-constructed lesson on actually doing a graph, constructing a graph and, so, the way that that worked was we had all the books out on the tables, and they went through, and they actually selected ones that they thought were easy to read and they were really different. Some of them I would not say were easy to read but when they actually explained it was because they used colour. So, going through all those tiny little

steps had come from all their work samples, and they (the students) were telling me what to do; "Put the ruler this way, no, put it horizontal" and I was literally just like a robot and doing things like, "Get your pencil here, start here" and as a group collectively they actually came up with, I guess, the perfect graph for them. It probably was not a perfect graph, but it made sense and it was all those ideas of what they thought was clear put together into one thing.

The following Grade 1 teacher's view illustrates the capability of the students as they developed graphing skills through implementing the IMS pedagogy through this process of invention, evaluation and refinement.

Really, you would not really expect Grade 1 to be able to accurately transfer data to a graph, that is a pretty impressive skill. But, there is (only) four, five, six kids that probably could not do that. I have taught (year) five/six and seen it has been a struggle for them, so, the fact that they are able to grasp that already at Grade 1 is going to be such a huge advantage as they move through (the year levels).

The Ecology Sequence was adapted for Grade 4, extending students to graphically represent organisms found in different habitats to open up more sophisticated adaptation discussions. This entailed students drawing a birds-eye view map of the different sample plots.
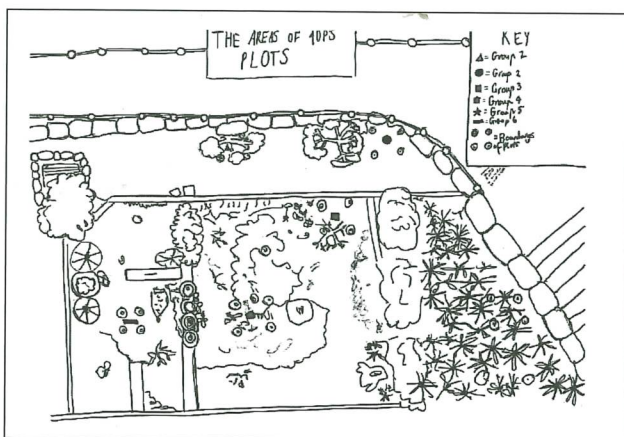

Figure 3. Grade 4 student's birds-eye view map of the school yard showing position of six different plots.

This task required an extended range of skills engaging students in visualising and spatial reasoning by requiring them to take a birds-eye perspective. Partitioning the school yard area into plots of reasonably accurate size drew upon the students' concepts of area and sub-division. In Figure 3 the student is able to illustrate the relative size and position of the six plots. The key provides a sophisticated system of locating, using symbols, the position of each plot. The mathematics involved here develops students' measurement and ratio and proportion skills as well as mapping skills. The notion of boundary is also depicted.

Astronomy Learning Sequence (Grade 1)

The Astronomy sequence, beginning with a shadows' investigation, followed on from the Year 1 Ecology sequence, providing students further opportunity to transfer their newly developed graphical skills to another authentic context. Students were asked the orienting question "What have you noticed about your shadows during the day?" Through whole class discussion students identified that shadows changed shape, length and direction. The teacher then posed a further question: "How can we track how they change?"

Students identified the need to record the shadow at different times during the day, making note of the length and direction. The need to measure arose naturally with no fixed solution or method provided by the teacher. This enabled varying recording systems to be negotiated and adopted by different classes. Some classes chose to record their shadow length with streamers, while others used blocks and their feet. Students suggested recording the data in a table and representing the collated recordings in a bar graph. This demonstrated students' recognition and use of bar graphs as an effective means of sharing and displaying data.

During the Building Consensus stage of the IMS pedagogy students identified the changing directionality of the sun influenced the direction and length of their shadow. They identified that at midday "the sun was directly above, and the shadow the shortest". Figure 4 is an example of a student's data table and bar graph of the change in shadow lengths for their measurements throughout the day. The graph shows their shadow in red and their partner's shadow in black. This was not modelled by the teacher, rather 'invented' by the student. This is indicative of how the learning design affords students the opportunity to demonstrate real understanding of the data. "I can tell the time with my body's shadow. In the middle part of the day the sun was above us, our shadows were smaller than others".

As a result, this student reasoned that their partner's data was "different, because although they have different feet lengths it must be the wrong measurements for the middle of the day" (Figure 4).
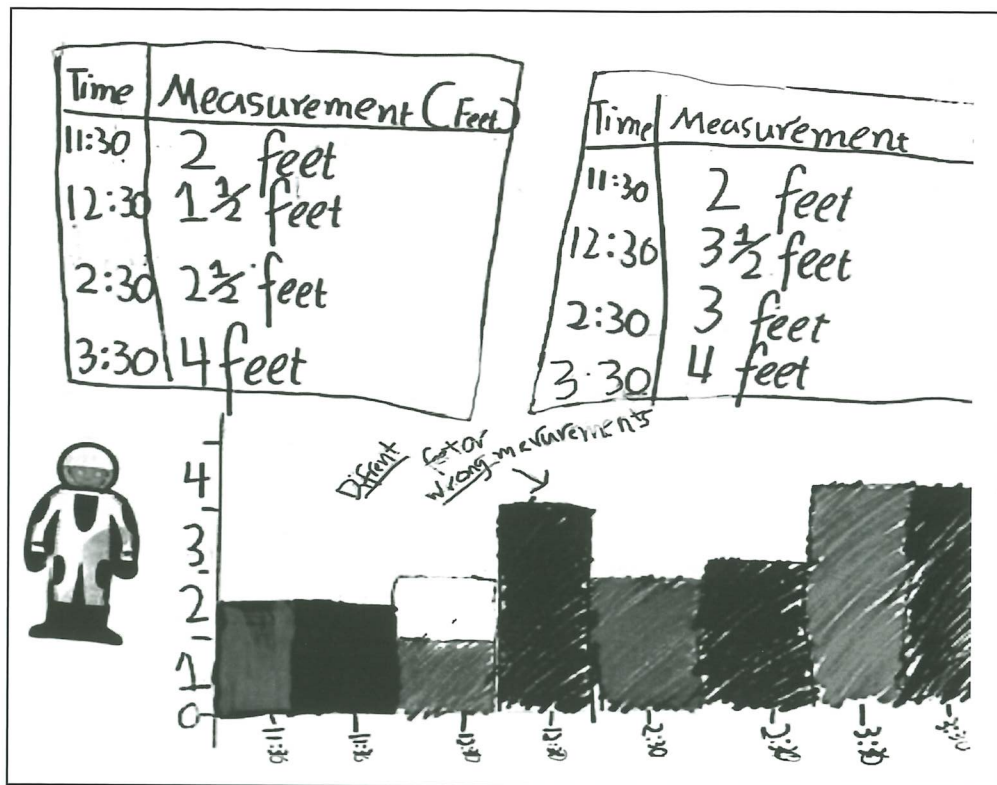
Figure 4. Grade 1 student's data table and graph comparing two students 'shadow lengths throughout the day.

Through class discussion the students realised that the 12.30 pm recording of three and a half feet was questionable as it didn't follow the pattern of getting smaller to midday.

In interview the student (red data Figure 4) suggested that they "would use a centimetres" if they were to conduct this investigation again as "it is an accurate way of recording". She further reasoned why the other data may be "the wrong measurement at 12:00" by stating the following;

Our shoe size might have been different, or the spot, the spot we were standing on could cause her shadow to be longer than mine. It would have been easier if we'd measured the same, same shoe size, because then we'd be measuring in the same size (unit).

This demonstrates the idea of formal measure and the need for it to be consistent to enable reliable comparison of data. Teachers commented on the advantages of learning mathematics through meaningful questions, providing both engagement and realisation of purpose.

Teacher: I think they (mathematics and science) do link naturally together. There were definitely times when there was more of a science focus or a maths focus and that could shift during

a lesson as well. Collecting data about things they're actually doing rather than hair colour or favourite lollies, is much more engaging and real for them (students) and I think it shows maths and how you use it in the real world rather than in a math lesson.

'Heating Up' learning sequence (Grade 3)

The Heating Up learning sequence was adapted from the *Primary Connections Units: Heating Up and Melting Moments* (Australian Academy of Science, 2020). The mathematical processes and practices involved data recording, temperature estimation and recording, timing with stop watches, comparison and ordering measures, and creating data tables and graphical representations. In one activity, the students were challenged to design and make their own cup to keep water hot. In groups, they tested and recorded the temperature of the water in their cup and a paper cup (the control) at consistent time intervals of two minutes. They then represented and compared the data to make mathematically justified claims about the effectiveness of their cup design. Students devised ways of showing temperature change over time.

Through the Building Consensus stage, by sharing their ideas and representations, the whole class agreed that a line graph was an effective way of comparing the effectiveness of their designed cup by 'showing change over time' (Figure 5). Students had prior
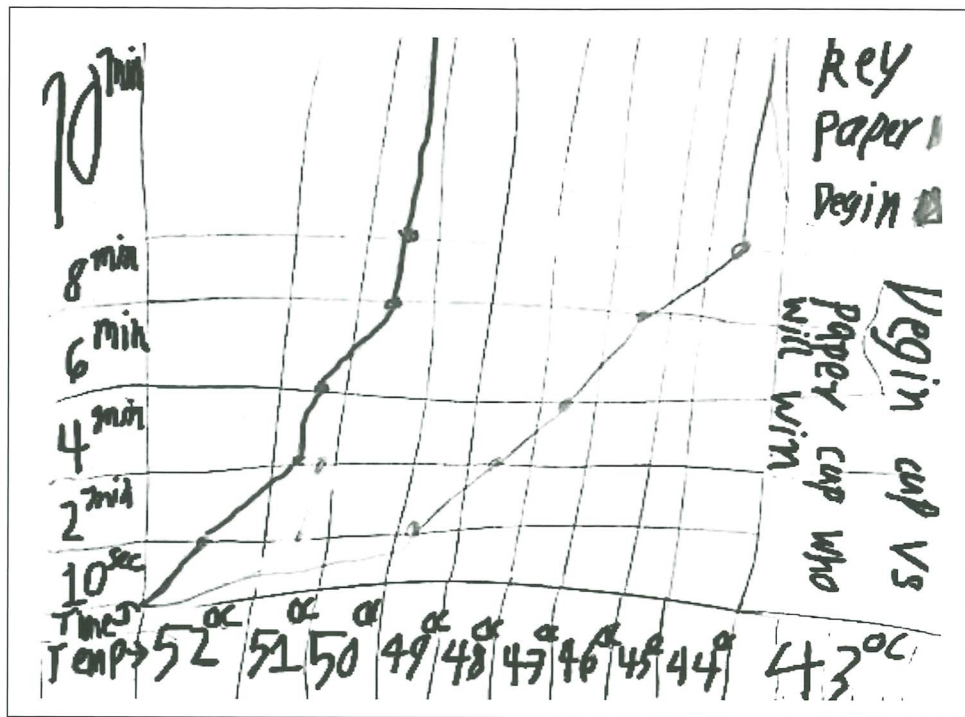
Figure 5. A Grade 3 student's representation of water temperature in two cups, their own cup design and a paper cup (control).

experience with constructing graphs with a horizontal and vertical axis to relate two different aspects; time and temperature. With teacher scaffolding the students suggested the idea of the line to join data points, showing the drop in temperature. Interestingly, the temperature was recorded on the horizontal axis in relatively accurate intervals. They then represented and compared the data to make mathematically justified claims about their cup design effectiveness.

The following excerpt from a teacher interview illustrates the value of the learning experiences for the students.

It was a link with graphing in particular, 'cause normally in maths you would teach it as a stand-alone topic… but to be able to use it with science… to be able to integrate it in with something, it shows the kids that it's real life. It's something that you're going to use—it's not just this thing that's by itself. It's integrated into other subjects as well and it can help us to sort of monitor the progress …I'm just thinking of the timing, when they had to time the heat or the cold water. And they had to work out how they were going to tally that or graph it, keep the data, collect the data and then what kind of graph they were going to use. So, it was good to hear them questioning and using temperatures, talking about Celsius degrees. All of those things came through the science experiment instead

of just being in a maths lesson where it doesn't mean anything. They got to actually use it and see it in action.

Implications for the mathematics classroom

The IMS project demonstrates a huge range of possibilities for mathematics learning supporting the *Australian Curriculum: Mathematics*, especially in light of the reasoning and problem-solving aspects of the Proficiencies (ACM, 2018). Students' learning experiences were illustrated by the examples: data modelling including graphical work, spatial skills, measurement including consistent units and standard units, invention of measurement processes (organism distribution, shadows, temperature).

The learning experiences also showed the development of mathematical practices well beyond expectations through this invention/comparison refinement process. As one teacher commented: "I think the students learnt more from each other than me". Nevertheless, the sequences are implemented with strong teacher guidance and the need for teacher flexibility/creative response to students' ideas. The implication of this is that teachers need to understand not only the mathematical concepts and procedures but also the underlying commonalities with scientific thinking and of mathematical practice, in order to

support deeper learning. Learning to work with data exploration and modelling was central to many lessons.

The project highlighted the advantages of situating the mathematics in meaningful problems: it made a real difference to student engagement, but also it taught students the underlying purposes of the mathematics—when and why particular mathematical conventions are important for identifying patterns and differences. There were challenges faced in the project's implementation, given the culture of mathematics teaching. Teachers were not always willing to give up mathematics time for this and to some extent they saw it as separate from the formal mathematics curriculum. From a curriculum sense, given the importance of mathematics in supporting science understanding, it may be appropriate to deal with it in science time (often not appropriately allocated in schools). We hope that the project has raised awareness of the possibilities of viewing mathematics and science synergistically, but we also raise questions about the influence of current traditions often limiting mathematics teaching to the mathematics 'timeslot' in schools.

References

Australian Academy of Science. (2020). *Primary Connections: Linking Science with Literacy. Resources and Pedagogies*. Retrieved from https://primaryconnections.org.au/resources-and-pedagogies

Australian Curriculum, Assessment and Reporting Authority (ACARA). (2018). *The Australian Curriculum: Mathematics*. Retrieved from http://www.australiancurriculum.edu.au/ Mathematics/ Curriculum/F–10

Lehrer, R., & Schauble, L. (2012). Seeding evolutionary thinking by engaging children in modeling its foundations. *Science Education, 96*(4), 701–724.

Lehrer, R., & Schauble, L. (2020). Stepping carefully: Thinking through the potential pitfalls of integrated STEM. *Journal for STEM Education Research*, 1–26.

Mulligan, J. (2015). Moving beyond basic numeracy: data modeling in the early years of schooling. *ZDM Mathematics Education, 47*(4), 653–663.

Tytler, R. (2019). New pedagogies for interdisciplinary STEM skills. *Education Matters*. https://www.educationmattersmag.com.au/new-pedagogies-for-interdisciplinary-stem-skills/

Tytler, R., Mulligan, J., Prain, V., White, P., Xu, L., Kirk, M., Nielsen, C., & Speldewinde, C. An interdisciplinary approach to primary school mathematics and science learning. *International Journal of Science Education*. doi:10.1080/09500693.2021.1946727